Evaluating Corporate Bonds with Complex Debt Structure

Dai, Tian-Shyr *   Wang, Chuan-Ju †   Liu, Liang-Chih ‡

Abstract

Most structural models of credit risk ignore the impacts caused by various properties of outstanding bonds and may carry out biased analyses against real world phenomena observed by empirical studies on debt heterogeneity. This paper analyzes the impacts of debt heterogeneity on the values of corporate securities and issuers' redemption policies by incorporating the following four facets of an issuer's debt structure: the leverage ratio, maturity structure, priority structure and covenant structure into a structural model. These complex analyses are achieved by constructing a novel quantitative framework, the multi-layer forest, to capture the contingent changes of an issuer's capital structure due to certain properties of that issuer's debt structure, like early redemption provisions embedded in callable bonds. Our work provides theoretical insights and concrete quantitative measurements on empirical phenomena, like the shapes of credit spread curves, the impacts of payment blockage and poison put covenant on other outstanding bonds of the same issuer, and the call delay phenomenon due to tax shield benefits and wealth transfer effect. This framework can also be applied to explore new phenomena that are hard to be empirically analyzed due to lack of data and illiquidity.

Keywords: Debt structure, maturity structure, priority structure, covenant structure, payment blockage, poison put, call delay

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*Department of Information and Finance Management, Institute of Finance and Institute of Information Management, National Chiao-Tung University, No.1001 Daxue Rd, Hsinchu City, Taiwan. E-mail: d88006@csie.ntu.edu.tw. Tel:+886-3-5712121 ext.57054. Fax:+886-3-5715544.

†Department of Computer Science, Taipei Municipal University of Education, No.1, Ai-Guo West Road, Taipei, Taiwan. E-mail: jerewang@gmail.com. Tel: +886-2-23113040 ext 8936

‡Institute of Finance, National Chiao-Tung University, No.1001 Daxue Rd., Hsinchu City, Taiwan. E-mail: tony919kimo@gmail.com.

1 Introduction

A corporate bond is not only a fundamental financing instrument for a firm to raise funds but one type of popular investment tools that are widely hold by institutional investors or fund managers. According to the reports of Securities Industry and Financial Markets Association (SIFMA), the amount of issuances (outstandings) in the US market grows from 343.7 billions (2126.5 billions) in 1996 to 1359.8 billions (9100.4 billions) in 2012.[1] This entails that a corporate bond is playing more important roles in capital market, and its prevalence further pushes the problem of how to properly evaluate those bonds into a lasting contested issue in both academic and practitioner communities. Regarding the effect of credit risk on bond values, bonds can intuitively be divided into two groups: default-free and defaultable bonds. A default-free bond, such as a treasury bond, can be separately evaluated from other simultaneously outstanding treasury bonds because the issuer's credit quality can be ignored.[2] Conversely, corporate bond holders suffer default risk because the issuing firm may fail to fulfill its due bond repayments. Numerous existing literature then accentuates the credit quality of an issuing firm as an essential determinant and further treats a bond value as a function of default triggers portrayed by corporate capital structure and the corresponding loss of the promised payments due to liquidation. While a large body of theoretical researches regard capital structure solely as a combination of equity and uniform debt, real world observations recognize debt heterogeneity in light of the empirical investigations by Rauh and Sufi (2010) and Colla et al. (2013) that an issuing firm actually relies on various types of bonds *simultaneously*, involving different maturities, seniorities, covenant restrictions, embedded options and so on. Such multidimensionality of corporate debt probably shapes default triggers in complicated ways that fail to be characterized by extant theoretical models of credit risk, but the recognition of debt heterogeneity indeed moves the topic of bond valuation in the empirically observed direction. Therefore, we address that, rather than being evaluated separately, corporate bonds issued by the same firm must be priced considering the presence of other simultaneously outstanding bonds with different properties. Any omissions of these observable information may lead to biased valuation results in contrast with empirical observations. Being as a complement to the deficiency of existing models, a theoretical framework is developed to model the multidimensionality of corporate debt.

The elementary sources of observable credit spreads on corporate bonds come chiefly from the presence of an issuer's default risk characterized by its capital structure and the corresponding loss of the promised payments due to liquidation. Hence, the features of each issuer's debt are crucial to the determination of bond and equity values. Rather than being treated as uniform, corporate debt is a structure with several facets woven by those simultaneously existing bonds: the leverage ratio, maturity structure, priority structure and covenant structure. The leverage ratio measures the proportion of a firm's debt to its asset and can be applied to estimate the firm's ability to fulfill its debt obligations. Generally, the credit rating of a firm is negatively related to its leverage ratio since a higher leverage ratio implies a higher level of debt obligations (compared to the firm asset value) and hence a higher default likelihood (Kisgen, 2006). The maturity structure governs the payment schedule of debt obligations. The empirical studies in Helwege and Turner (1999) and Huang and Zhang (2008) suggest that the term structure of credit spreads is usually upward-sloping, which implies short-term bonds suffer less default risk than long-term bonds of the same issuer. A possible explanation is that the repayments of short-term bonds would deteriorate the issuer's financial

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[1] See http://www.sifma.org/research/statistics.aspx

[2] Of course, it shall be under the assumption that sovereign risk is trivial.

status and increase the risk of long-term bonds. The priority structure determines the order of asset distribution to each claim holder once the firm is liquidated. Under the absolute priority rule, the payments to senior bond holders are satisfied in full before those to junior bond holders. Although the empirical studies in Bris et al. (2006) report no violation of this rule under the liquidation process associated with Chapter 7 of the U.S. bankruptcy code, the presence of short-term junior bonds may still deteriorate the effective priority of those previously issued long-term senior bonds as mentioned in Ingersoll (1987). Diamond (1993) and Park (2000) suggest that such priority deterioration problem can be avoided by prioritizing the debt structure as the way that makes short-term bonds be always senior to long-term bonds, but numerous exceptions are found and studied in Linn and Stock (2005). The covenant structure describes the constraints embedded in outstanding bonds of a given issuer to either protect the benefits to the holders of the previously issued bonds from the claim dilution problem or attenuate the conflicts of interest between bond and equity holders such as the asset substitution and underinvestment problem (Smith and Warner, 1979). One of the salient examples to mitigate the effective-priority-deterioration problem is the payment blockage covenant that allows senior bond holders to block scheduled payments to junior bond holders within the so-called blockage period to fulfill the repayments of senior bonds given the issuing firm defaults (Linn and Stock, 2005; Davydenko, 2007). Another example to alleviate asset substitution problem is the usage of the poison put covenant that can effectively deter equity holders from undertaking high-risk investments, such as mergers and acquisitions. Evaluating corporate bonds with covenant structure is becoming important since Billett et al. (2007) find that a firm with poor credit quality would be required to include more covenants in its new bond issuances. Following the features of these debt facets, we especially note that bonds with embedded options that drive early redemptions, such as call provisions, may result in potential changes of maturity structure and corresponding effective priority. Below, corporate debt is intentionally separated into the aforementioned four dimensions because they can be jointly observed by the participants in the bond market. Bond evaluation must adequately reflect these information.

To capture the relationship between credit risk and an issuer's debt structure, the theoretical framework of credit risk should be developed based on a credit risk model that can jointly characterize the aforementioned facets of debt structure. Traditional models like credit scoring systems (e.g. Altman (1968)) and expert judgement methods (e.g. Sinkey (2002)) depend either on experts' experiences or on the numbers recorded in financial statements. However, these models merely provide a coarse estimation for the creditworthiness of a given issuer. The features of debt structure are difficult to be incorporated into the analyses of bond and equity values. Reduced-form models are another popular credit risk model that directly characterizes a firm default process by calibrating market data but abstracts from the firm value dynamics and capital structure (Jarrow and Turnbull, 1995; Jarrow et al., 1997; Madan and Unal, 1998; Duffie and Singleton, 1999). Nevertheless, this type of models fails to identify the causes of financial distress so that it is hard to associate a credit event with debt structure. In contrast, structural models would jointly capture the capital structure as well as the evolution of firm asset value and further specify that credit events may result either from an issuer's inability to fulfill a due bond repayment or from its poor financial status (Merton, 1974; Black and Cox, 1976; Geske, 1977; Kim et al., 1993; Leland, 1994; Longstaff and Schwartz, 1995; Leland and Toft, 1996; Collin-Dufresne and Goldstein, 2001a). Fisher (1984) and Eom et al. (2004) report that existing structural models oversimplify an issuer's capital structure in order to make resulting mathematical models tractable but are deficient in capturing the multidimensionality of debt structure. Selectively ignoring any facets of debt structure that actually can be observed by market participants may enormously influence bond and equity values. That may be why numerous defects are

mentioned in the empirical studies on the validity of existing structural models (e.g. Collin-Dufresne et al.(2001b); Huang and Huang (2012)).[3] Two of the salient defects can be demonstrated through a simple example. Consider a firm that simultaneously issues three otherwise identical bonds with maturity 5-year, 10-year, and 20-year, respectively. **Fig. 1** displays the valuation results with or without the consideration of maturity structure woven by those simultaneously existing bonds. In **Fig. 1 (a)**, a conventional structural model that *separately* evaluates each bond without considering the existence of other outstanding bonds may overly underestimate the credit spreads for investment-grade issuers so that the valuation results (plotted in black dashed curve) are almost indifferent to that of default-free bonds as noted by Jones et al. (1984) and Kim et al. (1993). Besides, when the credit quality of the issuer deteriorates, a conventional structural model typically generates a hump-shaped (plotted in dark gray dashed) or a downward-sloping (plotted in light gray dashed) credit spread curve which is significantly inconsistent with the empirical results by Helwege and Turner (1999) and Huang and Zhang (2008) that most credit spread curves implied by the bonds issued on the same day by the same issuer are usually upward-sloping as illustrated in **Fig. 1 (b)**.[4] This primary experiment suggests that improving a structural model to *simultaneously* consider all outstanding bonds that weave the maturity structure may significantly fix the two salient defects of extant structural models.

Besides the oversimplification of an issuer's debt structure, the assumptions on financing future bond repayments will also affect the evolution of the firm asset value and the corresponding bond and equity values. One popular assumption pioneered by Black and Cox (1976) and Geske (1977) is that all a firm's bond repayments are financed by issuing new equities given the firm is solvent. Instead of issuing new equities, Leland and Toft (1996) proposes the framework of stationary debt structure, assuming that a firm will rollover all its maturing bonds to keep the lump sums of outstanding bond principals and coupon payments unchanged. The rollover gain and loss are totally absorbed by equity holders (He and Xiong, 2012). However, we note that these assumptions will force the models of credit spreads to generate the results that do not confirm by real world observations.

Our paper contributes to the literature on the structural models by developing a valuation framework that faithfully models the multidimensionality of corporate debt structure. As noted by Fisher (1984), Kim et al. (1993), Briys and De Varenne (1997) and Eom et al. (2004), our valuation framework complements to the deficiency of existing structural models in capturing the complexity of debt structure. Besides, as mentioned in Colla et al. (2013), this development will also complement the established empirical literature on corporate debt structure. Our framework will be verified to be consistent with the phenomena found in extant empirical researches and give theoretical insights to reasonably explain these findings. In addition, compared to equities that are liquidly traded in central exchanges, most corporate bonds are thinly traded in over-the-counter markets. Empirically analyzing the behaviors of participants in the bond market is much more difficult than analyzing those in

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[3]Collin-Dufresne et al. (2001b) find that the factors incorporated in extant structural models explain only about 25% of the variation in credit spreads. Huang and Huang (2012) indicate that structural models have difficulty in simultaneously explaining both the default probabilities and observable credit spreads, especially for investment-grade bonds.

[4]Jones et al. (1984) conclude that a (conventional) structural model is not an improvement over the model that evaluates default-free bonds for investment-grade issuers but seems to have incremental explanatory power for the bonds by speculative-grade issuers. However, most extant structural models predict hump-shaped and downward-sloping credit spread curves for a given speculative-grade issuer. The empirical results by Helwege and Turner (1999) show that, in primary market, over 80% of the credit spread curves implied by those equal-priority bonds issued on the same day by the same speculative-grade issuer are upward-sloping; over 60% of the credit spread curves for the cases in secondary market are also upward-sloping. The empirical investigation of the broader sample sets by Huang and Zhang (2008) displays that more than 80% of the credit spread curves for the cases of investment- and speculative-grade issuers are upward-sloping.
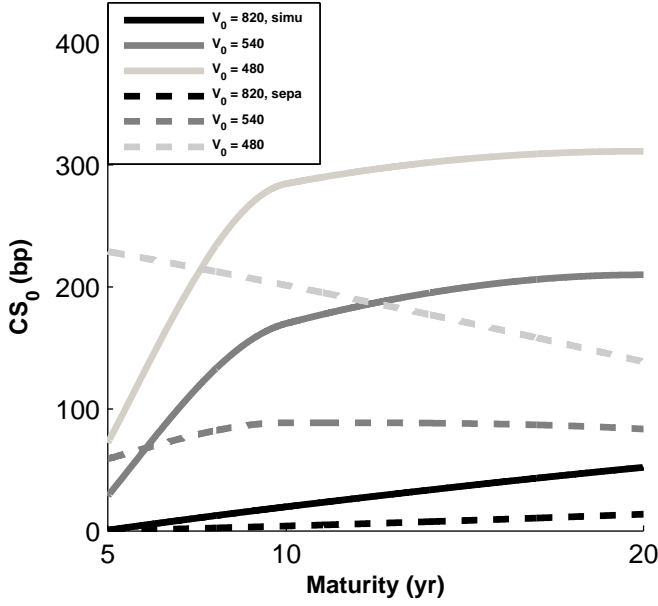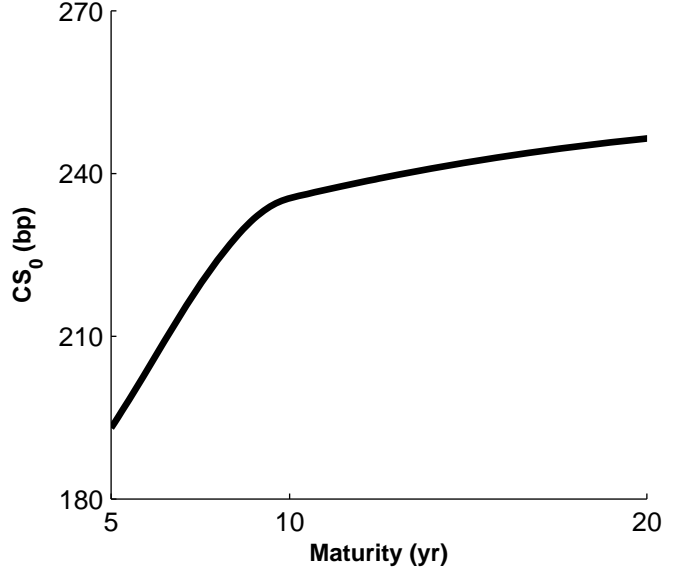
(a)   (b)

Figure 1: Evaluating Credit Spread Curves with(out) Considering the Maturity Structure. The numerical settings in panel (a) are as follows: the risk-free interest rate is 2%, the volatility of firm asset value is 0.2, the tax rate is 0.35 and the ratio of loss of firm value due to liquidation is 0.5. The solid and dashed curves denote the credit spreads implied by "simultaneously" and "separately" considering three equal-priority bonds of the same issuer, respectively. The black, dark gray and light gray curves denote that the creditworthiness of the issuing firm is proxied by a high (820) or a low (540 and 480) initial firm asset value, respectively. Panel (b) illustrates the credit spread curve implied by equal-priority bonds issued on the same day by a speculative-grade issuer, Arcelormittal SA Luxembourg. (Bond symbol: MT.AL, MT.AN and MT.AM.)

the equity market. We will present that the proposing framework can further give more concrete quantitative measurements for empirical findings rather than the statistical significant analyses and theoretically provide deductive inference of some properties that are hard to be empirically examined due to lack of data and illiquidity.

The rest of the paper is organized as follows. In Section 2, we describe the basic assumptions and the corresponding interpretation for our valuation framework. In Section 3, numerical implementation through lattice construction are elucidated, including the bino-trinomial structure to fit critical locations in our evaluation procedures, the structure of a multi-level forest to deal with potential early redemptions and the backward induction procedure considering a payment blockage covenant. In Section 4, we revisit the empirical results from previous literature through our numerical models. Section 5 concludes this paper.

2 Preliminaries

Developing a theoretical framework to associate observable credit spreads (or corporate bond prices) with corresponding corporate debt structures is an important future research work as mentioned in Colla et al. (2013). The debt structure can be described from four different facets: the leverage ratio, the maturity structure, the priority structure and the covenant structure. To model the interactions between corporate bond prices and the aforementioned facets, we construct a structural paradigm in credit risk modeling and employ the assumptions mainly from Ingersoll (1977a) and Attaoui and Poncet (2013) as follows. These assumptions can be separated into three categories: general assumptions sketching the behaviors of market participants and the rules generally obeyed by markets, model assumptions describing the underlying mathematical models of our valuation framework, and covenant assumptions enumerating the content or setting of call(or put) provisions and covenants examined in the later section.

The general market assumptions are

(A.1) Market participants (including equity and bond holders) prefer more wealth to less. Prices are determined in the market place such that perfect substitutes are valued identically.

(A.2) A manager of an issuing firm acts to maximize the benefits of equity holders rather than the overall firm value at the expense of bond holders subject to the restrictions included in the bonds. Similarly, bond holders will exercise their options to maximize their benefits.

(A.3) There are no transaction costs and all investors have equal access to information.

(A.4) Tax interest savings and bankruptcy costs are the only frictions in capital markets. A firm pays income tax at rate τ and this will lead to tax interest savings. On the other hand, a firm is immediately liquidated once it files for bankruptcy based on the Chapter 7 of the U.S. bankruptcy code. A constant fraction α of firm asset value is lost due to liquidation (e.g., lawyer and court fees). Note that tax interest savings and bankruptcy costs are the only benefits and drawbacks for a firm to use debt capitals and that both τ and α are within the range $[0, 1)$.

(A.5) When an issuing firm fails to fulfill its debt obligation, the firm files for bankruptcy and is immediately liquidated. The remaining asset is distributed according to absolute priority rule. The reorganization procedures in Chapter 11 proceedings like grace periods and subsequent debt renegotiations are not considered for simplicity.

The first three assumptions are from Ingersoll (1977a). To quantitatively analyze the call delay phenomenon for callable bonds (Ingersoll, 1977b), we introduce market frictions as in (A.4) rather than following the perfect market assumption.

The model assumptions are

(A.6) Uncertainty is formalized by the filtered complete probability space $(\Omega, F = \{\mathcal{F}_t, 0 \leq t < \infty\}, \mathcal{F}, \mathbf{Q})$. The processes of all discounted firm market values are martingales under the risk-neutral probability measure $\mathbf{Q}$ (Harrison and Kreps, 1979), and thus all contingent claims can be evaluated by using the risk-neutral valuation method. Ω

denotes a sample space, and $\mathcal{F}_t$ is an augmented filtration generated by z, a standard Brownian motion that governs the randomness of the following firm value dynamics:

$$dV_t = rV_t dt + \sigma V_t dz + C_t^I - C_t^O. \tag{1}$$

Here we follow Attaoui and Poncet (2013) by assuming that the firm value grows at the long-term average risk-free rate r since Ju and Ou-Yang (2006) suggest that the impact of stochastic interest rate can be negligible under this assumption. σ denotes the volatility. To faithfully capture the cash inflows due to raises of equity or debt capitals and cash outflows due to bond repayments or dividend payouts, our framework extends the firm value dynamics in Merton (1974) by jointly including the plus term C_t^I and the minus term C_t^O which represent the cash inflows and outflows that can be observed (or expected) at time $t = 0$ and will be realized at time $t > 0$.[5] Note that a firm files for bankruptcy and is liquidated once $V_t \leq C_t^O$ for all $t > 0$.

(A.7) A firm is assumed to issue multiple bonds: B_1, B_2, ... to comprise its debt structure. The i-th bond is assumed to mature at time T_i with annual coupon C_i and the face value F_i, where $T_1 < T_2 < T_3 \dots$. Coupons are assumed to be paid continuously for simplicity.

(A.8) To focus attention on the association between corporate bond valuation and different facets of debt structure, our framework keeps the investment policy of an issuing firm unchanged by setting the volatility of the firm asset value σ as a constant as in Fan and Sundaresan (2000). Besides, the dividend policy is also simplified by setting the dividend payout as 0. Actually, various dividend policies can be incorporated into this framework by properly tuning C_t^O.

The properties of corporate debt structure can be characterized through C_t^I and C_t^O. For example, given the firm asset value V_0, a certain time interval $[0, T]$ and other thing being equal, greater C_t^O for $t \in (0, T]$ implies a higher firm leverage ratio in this period. Similarly, given C_t^O for $t \in (0, T]$ and other thing being equal, low V_0 implies a bad financial status. Hence, the credit quality of a firm in our framework can intuitively be proxied either by the firm asset value or by the firm leverage ratio. To accomplish contingent claims analysis with analytical formulae, some literature imposes additional assumptions on C_t^I and C_t^O. Two popular assumptions are listed as follows:

(A.6-1) Black and Cox (1976) and Geske (1977) assume that bond repayments are financed by issuing new equities given that the firm is solvent. Specifically, bond repayments neither change the firm asset value nor add new outstanding bonds (i.e., $C_t^I - C_t^O \equiv 0$ for all $t \in (0, T]$ if the firm is solvent).

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[5]Under the risk-neutral valuation method, the firm value dynamics adopted by Merton (1974) can be adapted as

$$dV_t = (rV_t - C)dt + \sigma V_t dz,$$

where C represents the total dollar payouts by the firm per unit of time. Eq. (1) is identical to Merton (1974)'s dynamics when $C_t^I = 0$ and $C_t^O = Cdt$ for all t. The implication behind the setup of Eq. (1) and the firm value dynamics in many literature on contingent claims analysis is perfect liquidity mentioned especially by Jones et al. (1984). Perfect liquidity implies no risk of firm internal liquidity. Assumption (A.4), on the other hand, suggests perfect external market liquidity. Numerous studies investigate both a firm's solvency risk and internal liquidity risk, such as Davydenko (2007) and Gryglewicz (2011), whereas He and Xiong (2012) emphasize the risk of external market liquidity. To focus our attention on the association between the complexity of corporate debt structure and the value of each claim holders, our framework only considers solvency risk for simplicity.

(A.6-2) Leland and Toft (1996) propose the stationary debt structure assumption under which a firm will rollover all its maturing bonds to keep the lump sums of outstanding bond principals (LSP) and coupon payments (LSC) unchanged. That is, a solvent firm replacing every maturing bond by issuing a new outstanding bond with the same face value and coupon rate (i.e., $C_t^I - C_t^O \equiv -\texttt{LSC}$ for all $t \in (0, T]$ if the firm is solvent), and the rollover gain and loss are assumed to be totally absorbed by equity holders (He and Xiong, 2012).

Assumption **(A.6-1)** implicitly prevents an issuing firm from financing its bond repayments with its internal funds or assets. However, Eom et al. (2004) point out that such restriction is not typical, and that may be one of the reasons why Geske (1977)'s model underestimates bond spreads on average.[6] Lando (2004) also makes a similar comment that structural models with **(A.6-1)** may underestimate the spreads of long-term bonds. On the other hand, the empirical studies by Helwege and Turner (1999) and Huang and Zhang (2008) suggest that most credit spread curves implied by those bonds issued on the same day by the same firm are upward-sloping as illustrated in **Fig. 1 (b)**. Incorporating either **(A.6-1)** or **(A.6-2)** would generate different shapes of credit spread curves for those bonds that are newly issued. Specifically, incorporating **(A.6-1)** into a structural model to evaluate the five otherwise identical bonds with maturity 3-year, 6-year, 8-year, 12-year and 20-year would generate a downward-sloping credit spread curves as plotted by the gray curve in **Fig. 2 (a)**. This is because near-term equity value will be diluted to account for the fact that future debt obligations are repaid through equity financing. Thus, with **(A.6-1)** and the consideration of multiple bond issuances, short-term bonds are riskier because the firm will file bankruptcy once the equity value approaches to zero. Incorporating **(A.6-2)** would generate an odd-shaped credit spread curves as plotted by the gray curve in **Fig. 2 (b)**. This is mainly because the principal repayments to each holder of the aforementioned five outstanding bonds are assumed to be financed by issuing new bonds (i.e., identical bond with the same maturity), implying that multiple bonds will mature at the same future time t and would increase the burden of bond repayments and the credit spreads at time t. For example, the principal repayment of the bond that will mature at year 3 will be financed by issuing another 3-year bond that will mature at year 6. Thus two bonds (the 6-year bond and the 3-year bond issued at year 3) will mature at year 6 and lift the credit spread of the 6-year bond. It can be observed in **Fig. 2 (b)** that the strategy implied by **(A.6-2)** will irregularly lift the credit spreads at certain future times, say year 6 and year 12, and result in a "m"-shaped credit spread curves. Although the contingent claims analysis under Eq. (1) may be mathematically intractable as the arguments in Lando (2004), it is numerically solvable and can robustly generate upward-sloping credit spread curves as the black curve in **Fig. 2**.

To capture certain phenomena observed in the capital market, the following covenant assumptions are incorporated into our framework if necessary.

(A.9) Bond covenants are enforced once they are violated. The cases of covenant renegotiations and temporary (or permanent) covenant waiving are not considered.

(A.10) To avoid the deterioration in the effective priority of those previously issued long-term senior bonds due to the presence of short-term junior bonds as mentioned in Ingersoll

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[6]Geske (1977) evaluates the long-term junior zero coupon bond when the firm's debt structure is composed of a short-term senior and a long-term junior zero coupon bond. When asset sales are restricted, the equity holders would decide to repay the senior bond once they can raise enough equity capitals to meet the repayment. Otherwise, they decide to go bankruptcy. This repayment decision is regarded as an option, and the long-term junior bond and the corresponding equity can be evaluated by a compound options pricing formula.
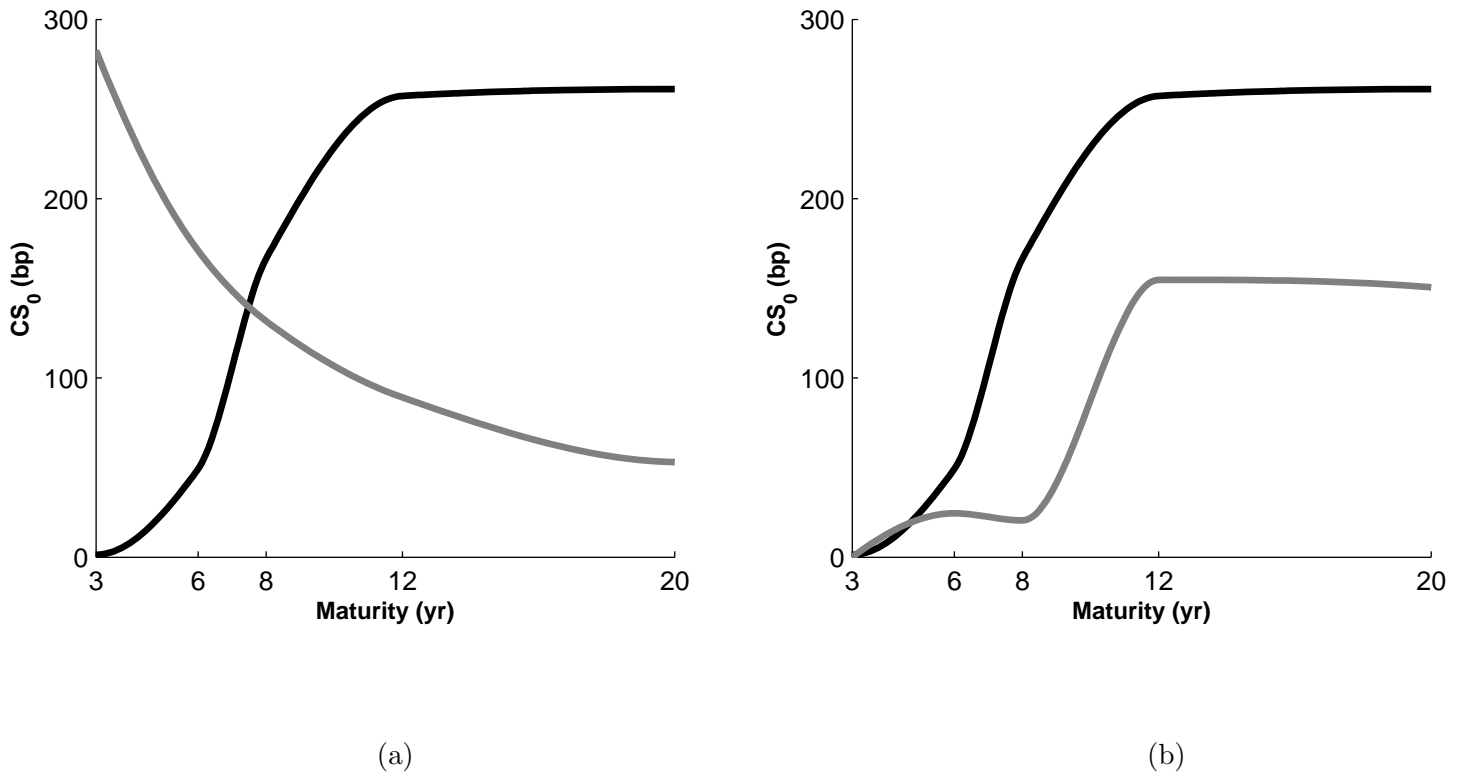
(a)   (b)

Figure 2: Evaluating Bonds under Different Payment Assumptions. This figure illustrated the credit spreads of five otherwise identical bonds of the same issuer with the maturity 3-year, 6-year, 8-year, 12-year and 20-year. The firm value V_0 is 900, the volatility σ is 0.22, the risk-free rate r is 2%, the tax rate τ is 0.35, the ratio of lost due to liquidation α is 0.5, and the face values of all bonds are 100. The x and the y axes denote the maturity in years and the corresponding credit spread in basis points, respectively. The black curves in both panels denote the term structure of credit spreads evaluated under **(A.6)**. The gray curves in panel (a) and (b) denote the term structures evaluated under **(A.6-1)** and **(A.6-2)**, respectively.

(1987), the payment blockage covenant can block scheduled payments to junior bond holders in order to fulfill the repayments to senior bond holders if the firm defaults on their payments (Linn and Stock, 2005; Davydenko, 2007). Specifically, if the issuing firm defaults at time t^*, $t^* > 0$, all the repayments to the junior bond holders during the so-called blockage period $[t^* - \iota, t^*]$ are blocked to satisfy the repayments to the senior bond holders. Note that the length of blockage period ι is usually ranged from 90 days to a year or more.[7]

(A.11) A poison put is an event-trigger covenant that protects bond holders from suffering the loss of credit quality due to the occurrence of event risks, such like leveraged buyout (LBO), by granting the holders the right to sell the bond back to the firm at a predetermined put price (PP). Here we assume that the early redemption is triggered

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[7]See http://www.uccstuff.com/CLASSNOTES/SubordinatedDebt.shtml. On the other hand, Cummings et al. (2009) mentions that the blockage period is usually less than 180 days. See also http://www.iflr.com/Article/2072881/IFLR-magazine/Back-to-basics-banking.html.

once the firm asset value declines and reaches the lump sum of the total bond principals.

(A.12) The effective call price (CP) of a callable bond is assumed to be the face value of the bond plus the accrued interest.[8] When the capital market is frictionless, Brennan and Schwartz (1977) and Ingersoll (1977a) argues that a callable bond shall be redeemed immediately once its market value soars and reaches its effective call price. We follow Longstaff and Tuckman (1994) by calling this strategy the "textbook policy". The call delay phenomenon is measured by observable "premiums over effective call prices" (PoCP); that is, the amount of an "about-to-call" bond market price above the CP of the callable bond.[9] On the other hand, the call protection period is a covenant that protects the bond holders from suffering reinvestment risk because of the premature redemption exercised by the corresponding bond issuers. The longer the period, the more protection for bond holders and the less value of the embedded call options.

The presence of equal or higher priority bonds may dilute the values of previously issued bonds (Fama and Miller, 1972), and the conflicts of interest between bond and equity holders arise from the behavior of a firm manager to maximize the equity value rather than the overall firm value as described in assumption **(A.2)** (Jensen and Meckling, 1976; Myers, 1977). Many bond covenants are designed to alleviate these problems (Smith and Warner, 1979), and the effect of a covenant on bond and equity values can be quantitatively estimated by comparing the values of all outstanding bonds and the corresponding equity given the presence or absence of the covenant. With assumption **(A.7)** and **(A.8)**, we will concentrate our attention on claim dilution, asset substitution and underinvestment discussed in Smith and Warner (1979). The deterioration in the effective priority of those previously issued long-term senior bonds due to the issuance of short-term junior bonds is an example of claim dilution and can be effectively alleviated by payment blockage covenants observed empirically by Linn and Stock (2005). On the other hand, for the fixed investment policy implied by a constant volatility of the firm asset value in Eq. (1), bond holders may suffer asset substitution or underinvestment problems given this policy is chosen to maximize the benefits of equity holders. Asset substitution comes from the fact that a manager has incentive to increase the equity value at the expense of the bond holders by exchanging the firm's low-risk assets to high-risk investments. One of the notable examples of high-risk investments is financial restructuring, such as LBO, and the presence of poison put covenants in the bonds of a target firm can significantly mitigate the asset substitution problem as the investigation by Cremers et al. (2007). Such effect of including poison put covenants can be quantitatively measured by evaluating the bidder's costs of raising additional bonds (to finance the LBO) and the value of the target bond holders with or without such covenants. Underinvestment arises when the firm manager forgoes a project with a positive net present value since the major part of the project return will be taken away by bond holders. Call provisions can efficiently attenuate this problem by limiting bond holders' returns. Such property of callability is confirmed by Bodie and Taggart (1978), Barnea et al. (1980), Kish and Livingston (1992), Crabbe and Helwege (1994), Sarkar (2004) and Jung et al. (2012) and can be well illustrated by the sensitivity of bond value to firm asset value (Acharya and Carpenter (2002)). However, call provisions may fail to mitigate underinvestment problems with the presence of market frictions (as in **(A.4)**) when the firm manager redeems bonds under the textbook policy. The textbook policy is empirically proven not to be the

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[8]Typically, the effective call price is defined as the call price determined in the contract plus the accrued interest. Here we assume that the call price determined in the contract is the face value of the callable bond.

[9]This measurement is adopted by Longstaff and Tuckman (1994) and King and Mauer (2000).

optimal redemption strategy for equity holders because positive `PoCP` are observable and widespread in the market. Note that callable bonds under the textbook policy imply no `PoCP`, and positive `PoCP` suggest that firm managers actually delay their timing of call (compare to timing implied by the textbook policy). Based on this specification and the assumption of **(A.4)** and **(A.7)**, the effects of tax shield benefits and wealth transfer to the holders of other outstanding bonds of the same issuer (Longstaff and Tuckman, 1994) on the call delay phenomenon are justified in later section.

3 Numerical Implementation

Under a structural model, all outstanding bonds and equity can be regarded as contingent claims on the issuing firm's asset value and can be evaluated by derivatives pricing methods once the firm asset value is characterized. A tree is a popular numerical technique to capture the firm value dynamics. It divides a certain time interval into equal-length time steps, and then bonds and the corresponding equity can be evaluated through backward induction on the tree. Pricing on trees is robust according to the statement by Duffie (1996) that such pricing results will converge to the theoretical values as the number of time steps approaches to infinity. However, even though the Duffie's statement is satisfied, node proliferation and the price oscillation phenomenon are still the problems that make pricing on trees infeasible.[10] Fortunately, Dai and Lyuu (2010)'s trinomial structure can avoid these problems. In addition, potential bond redemptions by exercising embedded call or put provisions may alter the maturity structure of a firm's debt and further change the credit quality of the existing bonds. Therefore, multiple-layer forests are developed to capture the influence of potential redemptions on corporate debt structure. The procedures of backward induction associated with maturity structure, priority structure and the concern about the payment blockage covenant are provided to complete the framework of pricing on trees.

3.1 Trees for the Lognormal Diffusion Process

A tree is a numerical method to portray the evolution of a stochastic process. It equally divides a certain time interval into several time steps and discretely specifies the value of the stochastic process at each time step. Derivatives on the stochastic process modeled by a tree can be priced through backward induction on that tree. Pricing results will converge to the theoretical values as the number of time steps approaches to infinity. For the lognormal diffusion process that is widely adopted as the firm value dynamics in financial literature, the CRR tree proposed by Cox et al. (1979) is the most prestigious binomial structure. The CRR tree discretely characterizes the lognormal diffusion process

$$dV_t = rV_t dt + \sigma V_t dz$$

through four parameters, u, d, P_u and P_d. u and d parameterize the state of the firm asset value, from the initial value V either up to Vu or down to Vd at the next time step. P_u and P_d parameterize the probability of up and down movement of the firm asset value for each time step. Given the interest rate r, the volatility of firm asset value σ and the time interval $[0, T]$ with n equal-length time steps

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[10]The number of nodes which have to be evaluated is liable to proliferate and become very large because the nodes on the tree do not recombine with the presence of discontinuous jumps (Hull, 2012). On the other hand, the pricing results from a tree may oscillate intensively because the nodes on the tree fail to coincide with the critical locations where the value function of the contingent claim is highly nonlinear (Figlewski and Gao, 1999).

Δt, $\Delta t = T/n$,

$$u = e^{\sigma\sqrt{\Delta t}}, \quad d = e^{-\sigma\sqrt{\Delta t}},$$
$$P_u = \frac{e^{r\Delta t} - d}{u - d}, \quad P_d = 1 - P_u.$$

Fig. 3(a) illustrates the CRR tree with $n = 2$. Note that the log-distance between any two vertically adjacent nodes on the CRR tree (e.g., node G and H) is $2\sigma\sqrt{\Delta t}$. If we define the notation $v(\phi)$ as the firm asset value on node ϕ and $f(t, V)$ as the discounted expected value of a contingent claim at time t when the firm asset value is V, then the discounted expected value of a contingent claim on node F can be expressed in the form of backward induction on the tree:

$$f(T/2, v(F)) \equiv e^{-r\Delta t}(P_u \times f(T, v(G)) + P_d \times f(T, v(H))). \tag{2}$$

To accurately describe Eq. (1) that involves discontinuous jumps due to cash inflows and outflows, the trinomial structures proposed by Dai and Lyuu (2010) are incorporated into the CRR tree to avoid the node proliferation problem. **Fig. 3(b1)** presents an example of a downward jump by a cash payout within the period $[0, T]$. Here we set $n = 2$ so that each time step $\Delta t = T/2$, and the jump occurs on node F at time $t = T/2$, denoted as $C_{T/2}^O$. Besides, we abbreviate $\ln(V'/V)$ as V-log-price of V' for convenience. If $(T/2)^-$ and $(T/2)^+$ represent the times immediately before and after the cash payout, $v(F) = V_{(T/2)^-} = Vu$ and $v(J) = V_{(T/2)^+} = Vu - C_{T/2}^O$. Then, in this example under the condition that $V_{(T/2)^+} = v(J)$, the mean of $v(J)$-log-price of V_T is μ, $\mu = (r - \sigma^2/2)\Delta t$, and the $v(J)$-log-price of $v(G)$, $v(H)$ and $v(I)$ is $\hat{\mu} + 2\sigma\sqrt{\Delta t}$, $\hat{\mu}$ and $\hat{\mu} - 2\sigma\sqrt{\Delta t}$. The trinomial structure can be used to connect an arbitrary node, J, to nodes placed as the CRR tree at the next time steps, G, H and I. The three branching probabilities of this structure p_u, p_m and p_d can be properly determined by solving

$$\begin{aligned} p_u\alpha + p_m\beta + p_d\gamma &= 0, \\ p_u(\alpha)^2 + p_m(\beta)^2 + p_d(\gamma)^2 &= \sigma^2\Delta t, \\ p_u + p_m + p_d &= 1, \end{aligned}$$

where

$$\begin{aligned} \alpha &\equiv \hat{\mu} + 2\sigma\sqrt{\Delta t} - \mu = \beta + 2\sigma\sqrt{\Delta t}, \\ \beta &\equiv \hat{\mu} - \mu, \\ \gamma &\equiv \hat{\mu} - 2\sigma\sqrt{\Delta t} - \mu = \beta - 2\sigma\sqrt{\Delta t}. \end{aligned}$$

The discounted expected value of a contingent claim on node J can then be expressed in the form of backward induction on the trinomial structure:

$$f((T/2)^+, v(J)) \equiv e^{-r\Delta t}(p_u \times f(T, v(G)) + p_m \times f(T, v(H)) + p_d \times f(T, v(I))). \tag{3}$$

Besides the node proliferation problem, Dai and Lyuu (2010)'s trinomial structure can also alleviate the price oscillation phenomenon. For the evaluation of corporate bonds and the corresponding equity, the critical location is the default boundary, i.e., $V_t = C_t^O$ for $t \in (0, T]$. The trinomial structure can

easily connect arbitrary nodes to those nodes of a CRR tree that has the price level to coincide with the default boundary, as illustrated in **Fig. 3(b2)**. The dashed trinomial structures can connect arbitrary nodes at time $T/3$, J and K, to those nodes of the CRR tree that is constructed originally from node L, $\upsilon(L) = V^* = C_T^O$.

3.2 Multiple-Layer Forests

Structural models implemented by trees are widely studied in past researches (e.g. Broadie and Kaya (2007); Dai et al. (2012)). However, developing a tree to model a debt structure with callable (putable) bonds is an intractable problem and has not been satisfactorily solved. This is because contingent redemptions due to the exercise of call (put) provisions will change the schedule of bond repayment and the future evolution of the firm asset value. Consequently, the values of corporate securities on the firm value dynamics with contingent redemptions is hard to be evaluated by a single tree.

To capture the contingent changes of the debt structure due to early redemptions, a multi-layer forest is developed as illustrated in **Fig. 4**. This forest is composed of several trees, and each tree captures the evolution of the firm asset value under one possible execution state of call (put) options embedded in outstanding bonds. Exercising an option to redeem a bond will change the execution state and cause the firm asset value to move from one tree, says a upper plain tree in **Fig. 4**, to another tree, says a lower plain tree in the same figure. The core idea is demonstrated by taking advantage of a hypothetical 7-time-step forest in **Fig. 4**. The firm is assumed to issue three zero-coupon bonds B_1, B_2 and B_3 with maturity $2T/7$, $5T/7$ and T and face values F_1, F_2 and F_3. B_2 is a callable bond with call price K_c and the other two bonds are straight bonds.[11] The upper plain and the lower plane trees model the firm value dynamics under two execution states: B_2 is not called yet and B_2 is already called, respectively. Specifically, in the upper plane tree, the firm asset value will jump down by F_1 (or F_2) at time $2T/7$ (or $5T/7$) due to the repayment of B_1 (or B_2). The firm defaults if it can not fulfill the due bond repayments: F_1 at time $2T/7$, F_2 at time $5T/7$, and F_3 at time T; nodes I, J, and K are decided to match these three critical locations to avoid the price oscillation problem. To match the critical locations and model the value jump without incurring uncombined tree structure, the trinomial structures (plotted in dash lines) by Dai and Lyuu (2010) are incorporated to adjust the tree structure, and the detail implementation is surveyed in **Fig. 3 (b)**. Therefore, given the condition that B_2 is not called, the values of equity and three outstanding bonds at each node on the upper plane tree can be evaluated by applying the backward induction method on the upper plain tree. Similarly, the lower plain tree models the firm value dynamics with two outstanding bonds: B_1 and B_3; the absence of the callable bond B_2 on the lower plain tree is due to the premise that B_2 is already called. The trinomial structures are incorporated into the tree to model the value jump F_1 at time $2T/7$ and to match the critical locations at nodes M and N. The values of equity, B_1 and B_3 at each node on the lower plain tree can also be evaluated by applying the backward induction method on the lower plain tree.

Recall that a firm will decide whether to redeem B_2 to maximize the equity value as mentioned in **(A.1)** and **(A.2)**, and the equity values under these two decisions can be evaluated by this two-layer forest. Take node U at time $3T/7$ for example. The equity value at node U given that B_2 is not redeemed, denoted as E', can be evaluated by directly applying the backward induction on the upper plain tree. On the other hand, redeeming B_2 will make the firm value jump from node U downward to node W; that is, $\upsilon(U) - K_c = \upsilon(W)$, where $\upsilon(\phi)$ denotes the firm asset value on node ϕ. Then,

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[11]The same forest structure can be applied to model a debt structure with a putable bond.

B_2 will be removed from the debt structure, and the outgoing branches from node W should connect to certain nodes, say X, Y, and Z, on the lower plain tree. This outgoing trinomial structure is constructed by the method proposed in **Fig. 3 (b1)**. The equity value given that B_2 is redeemed, denoted as E'', can be evaluated by calculating the expected discount equity values of nodes X, Y, and Z. If $E'' > E'$, the firm redeems B_2, and the values of B_1 and B_3 on node W are evaluated by applying the backward induction from node X, Y, and Z. Otherwise, the firm does not redeem B_2, and the values of equity and all bonds on node U are evaluated by applying the backward induction from U's successor node S and T. Note that the above option-execution decision will be applied to all the nodes on the upper plain tree prior to the maturity of B_2 (i.e., 5T/7). Besides, the evaluation of equity and all outstanding bonds are influenced by the call decision, and this property favors the analysis of whether the call delay phenomenon is caused by the wealth transfer to different claim holders.

3.3 Pricing on Trees through Backward Induction

When evaluating corporate bonds and the corresponding equity under a structural model with trees, the procedures of backward induction associate the values of each claim holder with the characteristics of the issuer's debt structure. Recall that the equity value can be regarded as the residual claim on the firm asset value, whereas the bond values are the function of default triggers and the corresponding loss of promised payments due to liquidation. On one hand, the leverage ratios and maturity structure determine the timing of default triggers. Potential changes of maturity structure driven by early redemptions may lead to potential deterioration in the credit quality of existing bonds. On the other hand, when the firm is liquidated, the seniority of bonds guarantees that the payments to senior bonds holders are satisfied in full before that for junior bond holders. Moreover, the holders of the previously issued senior bonds are granted the right to block the scheduled bond payments to junior bond holders. If the firm fails to fulfill the repayment of the senior bond, the payments to the junior bond holders occurred during the blockage period are forced to return to the senior bond holders. The payment blockage covenant appears to have salient valuation effect on bond prices considering both maturity and priority structure of corporate debt as discussed in Linn and Stock (2005) but is never implemented in structural models to the best of our knowledge.[12] Below, a generic case is provided to illustrate the backward induction considering the presence of payment blockage covenants.

A firm is assumed to issue two straight bonds B_1 and B_2 with maturity T_1 and T_2, face value F_1 and F_2 and annual coupon rate C_1 and C_2. Let $T_1 < T_2$ as in (**A.7**) and B_1 be junior than B_2, denoted as $B_1 \prec B_2$. The after-tax cash payments C_t^O can then be summarized as

$$
C_t^O = \begin{cases}
0 & \text{, if } t = 0, \\
(1-\tau)(C_1 + C_2)\Delta t & \text{, if } 0 < t < T_1, \\
F_1 + (1-\tau)(C_1 + C_2)\Delta t & \text{, if } t = T_1, \\
(1-\tau)C_2\Delta t & \text{, if } T_1 < t < T_2, \\
F_2 + (1-\tau)C_2\Delta t & \text{, if } t = T_2.
\end{cases}
\tag{4}
$$

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[12]The payment blockage covenants are especially highlighted to alleviate possible claim dilution for the holders of previously issued long-term senior bonds with the presence of short-term junior bonds. Extant structural models intentionally ignore the existence of this covenant by properly simplifying the corporate debt structure. For example, Geske (1977) specifies the case that a firm issues a short-term senior and a long-term junior bond. Hackbarth and Mauer (2012) and Attaoui and Poncet (2013) focus on the debt structure composed of perpetual and same finite maturity bonds, respectively

Note that Δt is the length of each time step on the tree. We also suppose $C_t^I = 0$ for all $t \in [0, T_2]$ here for simplicity. According to **(A.10)**, the bond repayments to B_1 holder during the blockage period $[t^* - \iota, t^*]$ are blocked to satisfy the repayments to B_2 holder given that the firm defaults at time t^*, $t^* > 0$. We then define $\texttt{BP}(t)$ as the value of "blocked payments" at time t; that is, the value of total repayments to B_1 holder during the blockage period at time t. Besides, we define $\texttt{UP}(t)$ as the value of "unblocked payments" at time t; that is, the value of total repayments to B_1 holder during the time interval $[t^* - \iota - \Delta t, t^* - \iota]$ at time t. The repayments are unblocked because the repayments to B_1 holder during the time interval $[t^* - \iota - \Delta t, t^* - \iota]$ are not in the blockage period so that B_1 holder are guaranteed to receive these payments even if the firm defaults at time t^*. For convenience, we also define $\texttt{PV}_i(t)$ as a value representing the discounted value of all future unpaid payments to B_i holder at time t, where $i = 1$ or 2 here. The values of equity, B_1 and B_2 can be evaluated by the following backward induction equations:

(Case 1): At B_2's maturity date T_2

$$E(T_2, V) = \begin{cases} V - C_{T_2}^O & , \text{if } V \geq C_{T_2}^O, \\ 0 & , \text{otherwise,} \end{cases}$$

$$B_2(T_2, V) = \begin{cases} F_2 + C_2 \Delta t & , \text{if } V \geq C_{T_2}^O, \\ \min(\texttt{BP}(T_2) + (1 - \alpha)V, F_2 + C_2 \Delta t) & , \text{if } V < C_{T_2}^O, \end{cases}$$

and

$$B_1(T_2, V) = \begin{cases} \texttt{BP}(T_2) + \texttt{UP}(T_2) & , \text{if } V \geq C_{T_2}^O, \\ \texttt{UP}(T_2) + \max(\texttt{BP}(T_2) + (1 - \alpha)V - F_2 - C_2 \Delta t, 0) & , \text{if } V < C_{T_2}^O. \end{cases} \quad (5)$$

Note that the firm is liquidated at time T_2 if the firm asset value is less than the payout $C_{T_2}^O$. The residual value $(1 - \alpha)V$ and the blocked payment $\texttt{BP}(T_2)$ are then used to satisfy the repayment to B_2 holder, $F_2 + C_2 \Delta t$. If the repayments to B_1 holder are not blocked by B_2 holder given the firm defaults at time T_2 (i.e., $T_1 \notin [T_2 - \iota - \Delta t, T_2 - \Delta t]$), then $\texttt{BP}(T_2) = 0$ and $B_1(T_2, V) = 0$ (i.e., $(1 - \alpha)V < F_2 + C_2 \Delta t$).

For convenience, let t^- and t^+ denote the times immediately before and after the bond repayment time t. Similarly, V^- and V^+ denote the firm asset value at time t^- and t^+, respectively. If the firm asset value at time t^- is less than the payout C_t^O, the firm will be liquidated. The blocked payments and the residual asset value $(1 - \alpha)V^-$ are first used to satisfy the repayment to B_2 holder, $\texttt{PV}_2(t^-)$. The remaining asset then distributes to B_1 holders. Otherwise, B_1 and B_2 holders are repaid because the firm survives and continues its operation. The value of a security at time t^+ (i.e., $E(t^+, V^+)$ or $B_1(t^+, V^+)$ or $B_2(t^+, V^+)$ here) can be evaluated as the expected discounted values of that security at the next time step as Eq. (2) and Eq. (3). The values of these securities at time t^- depend on whether the issuing firm is default or not and are evaluated by the following equations:

(Case 2): At any time t prior to T_2

$$E(t^-, V^-) = \begin{cases} E(t^+, V^+) & , \text{if } V^- > C_t^O, \\ 0 & , \text{otherwise,} \end{cases}$$

$$B_2(t^-, V^-) = \begin{cases} B_2(t^+, V^+) + C_2 \Delta t & \text{, if } V^- > C_t^O, \\ \min(\text{BP}(t) + (1-\alpha)V^-, \text{PV}_2(t^-)) & \text{, if } V^- \leq C_t^O, \end{cases} \tag{6}$$

and

$$B_1(t^-, V^-) = \begin{cases} \text{UP}(t) + B_1(t^+, V^+) & \text{, if } V^- > C_t^O, \\ \text{UP}(t) + \max(\text{BP}(t) + (1-\alpha)V^- - \text{PV}_2(t^-), 0) & \text{, if } V^- \leq C_t^O. \end{cases} \tag{7}$$

$B_1 \succ B_2$ is a simpler case because $\text{BP}(t) = \text{UP}(t) = 0$ if $t > T_1$. Then we can drop **Eq. (5)** from **(Case 1)** and exchange the subscripts of the terms B_2 and PV_2 in **Eq. (6)** and the subscripts of the terms B_1 and PV_2 in **Eq. (7)**. To deal with discrete coupon payments, we need to modify **Eq. (4)** and assign the payments on B_1 and B_2 repayment dates. **(Case 2)** will then be separated into 3 subcases. Two of them tackle the backward induction for either B_1 or B_2 coupon payment dates, and these procedures are similar to those above. The other tackles the backward induction for neither B_1 nor B_2 coupon payment dates, and this procedure drop the plus term of coupon payments. By adding more equations into **(Case 1)** and **(Case 2)**, the backward induction can also handle the scenario that a firm issues more than two bonds. Furthermore, potential leverage increasing by issuing new bonds can be easily dealt with by properly adjusting C_t^I and C_t^O in the procedure of backward induction. For example, the firm now issues a zero-coupon bond B_3 with face value F_3 and maturity T_3, $T_3 > T_2$ and $B_3 \prec B_1 \prec B_2$. Suppose the firm raises $B_3(0, V)$ dollars and half of the funds is for share repurchase. Then, $C_0^I - C_0^O = 0.5B_3(0, V)$ and $C_{T_3}^O = F_3$. In addition, the backward induction for potential early redemptions can be implemented on a multi-layer forest without difficulty once the execution strategies of call or put provisions are specified by predetermining redemption policies.
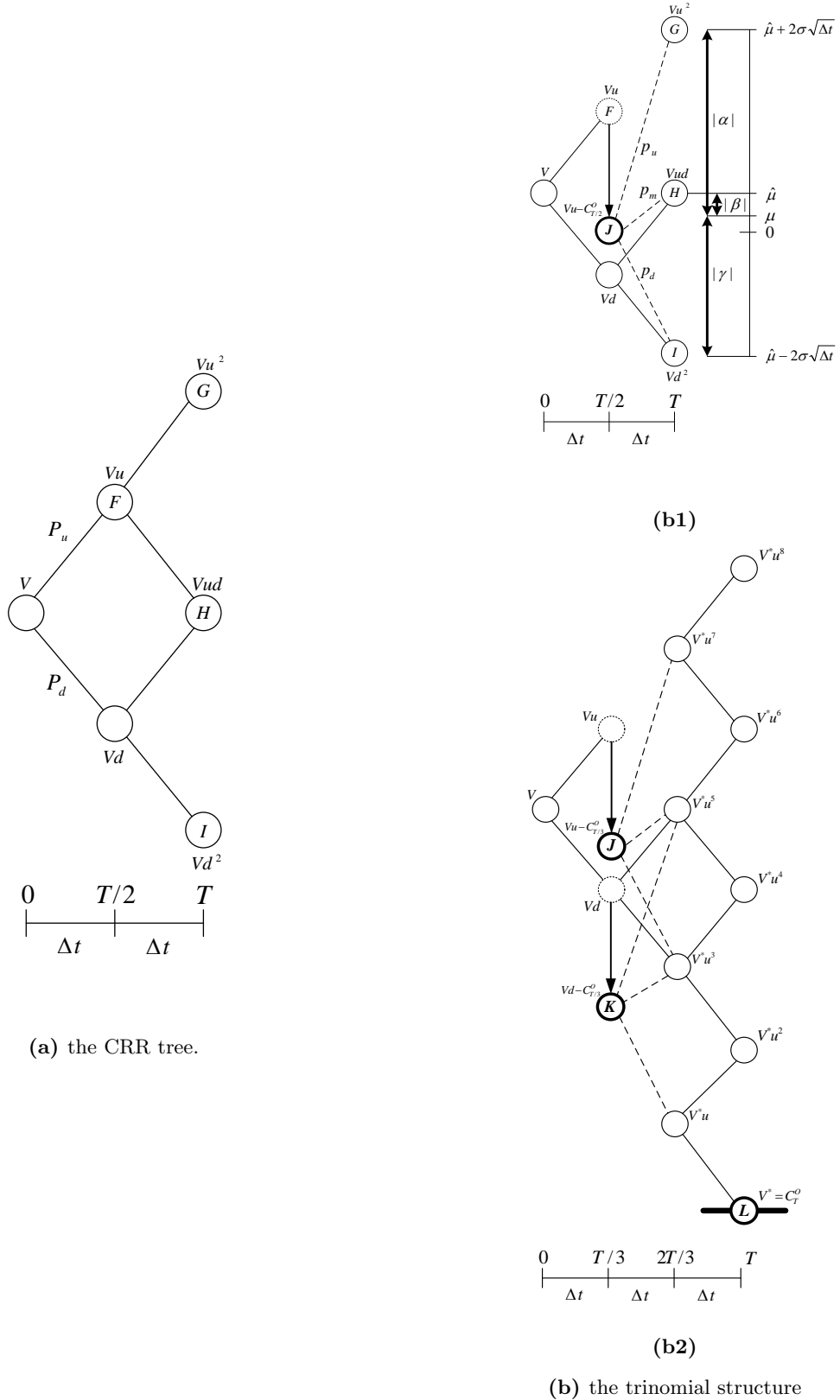
(b1)

(a) the CRR tree.

(b2)

(b) the trinomial structure

Figure 3: Trees for the Lognormal Diffusion Process. Panel (a) illustrates the CRR tree with the parameters u, d, P_u and P_d. u and d govern the state of firm asset value; P_u and P_d represent the branching probabilities of the firm asset value to move up and down. Panel (b1) presents the trinomial structure with the branching probabilities p_u, p_m and p_d that are determined by the parameters μ, $\hat{\mu}$, α, β and γ. This structure can avoid the node proliferation problem due to discontinuous jumps. Panel (b2) shows that the trinomial structures can alleviate the price oscillation phenomenon, because they can connect arbitrary nodes to those nodes of a CRR tree that has a node L or price level to coincide with the critical location plotted by the thick black line.
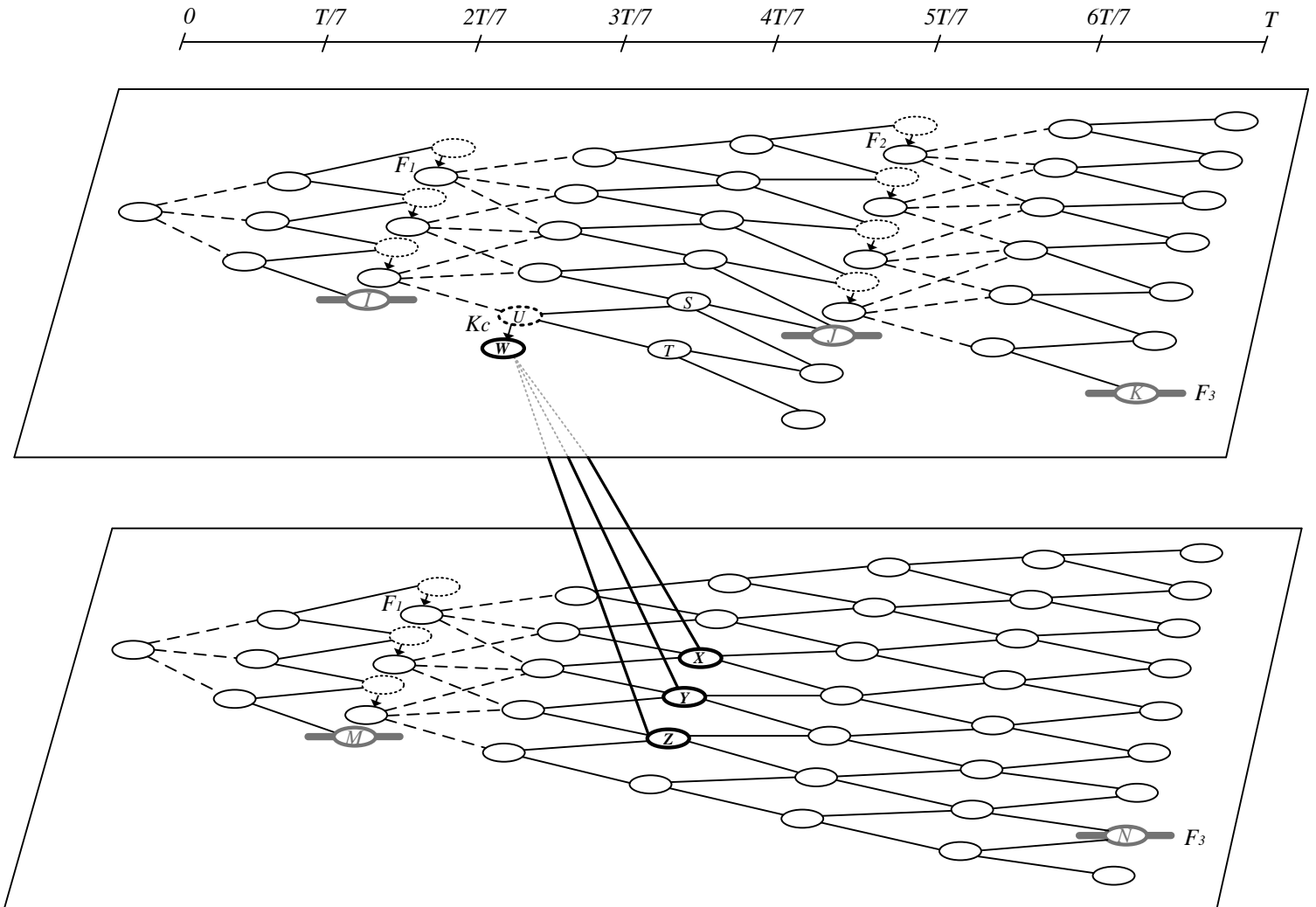
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Figure 4: A Two-Layer Forest. The evolution of the firm asset value with three outstanding bonds (i.e. B_1, B_2, and B_3) is modeled by the two-layer forest that is composed of two trees; the upper plain and the lower plain trees model the evolution of the firm asset value given that B_2 is not called and is already called, respectively. The CRR binomial structure is plotted by thin solid lines and the trinomial structure of Dai and Lyuu (2010) is plotted by dashed lines. Node I, J, K, M and N are decided to exactly match the default boundaries — the level of the firm value to match the due bonds. For example, $v(K) = v(N) = F_3$.

4 Empirical Implications

Abundant theories and corresponding elucidatory pros and cons according to empirical data are well documented by financial literature. Among these studies regarding credit risk modeling, structural models construct the linkage between corporate bond evaluation and capital structure but specify only fractional characteristics of a firm's debt for simplicity. Such simplification like homogeneous class of debt or identical bond maturity may both lose the generality and particularity of a firm's debt as the exploration by Rauh and Sufi (2010) and Colla et al. (2013) and partly violate the observable phenomenon. This section focuses on three scenarios based on four facets of debt structure — the leverage ratio (L), maturity structure (M), priority structure (P) and covenant structure (C) — as illustrated in **Table 1**. In scenario 1, we examine the valuation effect on previously issued bonds when a firm's debt structure is altered, including bond replacement as well as leverage increasing by issuing different type of bonds. In scenario 2, the function of callability provisions implied by different redemption policies are investigated to accentuate the optimal redemption policy in the capital market with frictions. Besides, our numerical results bridge the gap between the call delay phenomenon explained by the optimal redemption policy and observed by empirical data. In scenario 3, the protection through poison put covenants for the target bond holders and the corresponding effect on bidders' financing costs to leverage buyouts can be well illustrated by our model in line with empirical findings. In the following subsection, most of the parameter values in numerical experiments, such as the volatility of firm asset value σ, tax rate τ and bankruptcy cost α, are chosen from Leland (1994). The definition of the firm leverage ratio follows Collin-Dufresne et al. (2001b). A initial firm value V_0 is regarded as a proxy of the firm's credit quality.

Scenarios \ Facets	L	M	P	C
1	✓	✓	✓	✓
2		✓		✓
3		✓	✓	✓

Table 1: Three Scenarios Based on Empirical Studies and the Corresponding Facets of Debt Structure We Concentrate on. Three scenarios based on empirical studies will be discussed in the following subsections. We will focus on different facets of debt structure in each scenario. In this table, "L", "M", "P" and "C" represent the leverage ratio, maturity structure, priority structure and covenant structure, respectively. In scenario 1, valuation effect on existing bonds due to bond replacement as well as leverage increasing by issuing different type of bonds are discussed. Four facets of debt structure are concurrently considered in this scenario. In scenario 2, issuers' call policy and the corresponding alteration of maturity structure are investigated. The presence of multiple bond holders and call protection may greatly influence issuers' call policy. Thus, the facets of "M" and "C" are considered in this scenario. In scenario 3, the cost of leverage buyouts with or without the presence of poison put covenants in target firm's bonds are studied. Potential triggers of put may change the bidder's maturity structure and further deteriorate the priority of those bonds that the bidder issues to accomplish leverage buyouts. The facets of "M", "P" and "C" are considered in this scenario.

4.1 The Valuation Effect on Bonds Due to the Change of Debt Structure

The spirit of structural paradigm in credit risk modeling is to catch the idea that a firm incurs default when its leverage ratio approaches to unity. Hence, structural models predict that credit spreads increase with firm leverage ratios. Based on this idea, Collin-Dufresne and Goldstein (2001a) propose a model to further illuminate the viewpoint that the observed bond prices contain the information not only about the issuer's current leverage but also about the investors' expectations about future leverage. Such understanding has been well consolidated by the subsequent empirical studies of Collin-Dufresne et al. (2001b) and Flannery et al. (2012). Our framework can also generate the results in accord with the intuition shaped by extant theories and empirical investigations. For example, given a debt structure which is composed of an 8-year bank debenture (BD), a 12-year senior debenture (SD) and a 20-year long-term debenture (LTD) with priority BD $\succ$ SD $\succ$ LTD, the impact of bond issuance on these previously issued bonds is examined through our numerical model that simultaneously evaluates all of these bonds. As the results displayed in **Fig. 5 (a)** and **(b)**, both the increase in the contemporaneous and future leverage ratio raise the contemporaneous credit spreads of the firm.[13] Focusing on **Fig. 5 (a)**, we find that, given a firm's credit quality, the dilution effect of a short-term (high-priority) bond issuance is greater than that of a long-term (low-priority) bond issuance on the existing bonds. Besides, the longer the maturity of the bond issuance the more pronounced the difference in the dilution effect caused by new bond issuances with different seniorities, because the value of bond seniorities is greater with bond maturity. Based on this results, we infer that bond covenants are designed to force a firm to issue less bonds or issue longer maturity and lower priority bonds when the credit quality of the firm deteriorates.[14]

Besides the overall valuation effect on previously issued bonds due to new bond issuances, Linn and Stock (2005) specifies the impact of a junior debenture (JD) issuance on the previously issued SD. Their results empirically confirm the phenomenon that the replacement of the existing BD by the new JD decreases the spreads of the SD the larger the replace size, for the reason that the relative priority of the SD could be improved. Linn and Stock further emphasize that the magnitude of such impact is on average the same when the firm has good credit quality despite the maturity spectrum of the JD (shorter or longer than the maturity of the previously issued SD), but such difference becomes pronounced when its credit quality deteriorates, even though the SD holder has the right to block the scheduled repayments to the JD holder. With the example that a firm's debt structure is comprised by three types of bonds: bank debenture, senior debenture and long-term debenture, our numerical pricing framework can also generate the results in line with Linn and Stock (2005), as displayed in **Fig. 6**. In this numerical example, a JD is issued to replace the otherwise identical BD. Both **Fig. 6(a)** and **(b)** show that the spreads of SD decrease (denoted as negative ΔCS_0^{senior}) once the BD is replaced despite the Relative Maturity of the JD. The main difference between the results in **Fig. 6(a)** and **(b)** is the concern about the payment blockage covenant. In **Fig. 6(a1)** and **(a2)**, if the payment blockage covenant is ignored, the fact that the JD matures before or after SD (denoted as negative or positive Relative Maturity) leads to significant difference between the average value of all ΔCS_0^{senior} before the maturity of SD and that after the maturity of SD despite the firm's credit

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[13]We follow the Flannery et al. (2012)'s criteria that the future leverage is the leverage expected by investors within one year ahead from now.

[14]As a matter of fact, according to Guedes and Opler (1996), "large firm with investment grade credit ratings typically borrow at short end and at long end of the maturity spectrum, while firms with speculative grade credit ratings typically borrow in the middle of the maturity spectrum. This pattern is consistent with the theory that risky firms do not issue short-term debt in order to avoid inefficient liquidation, but are screened out of the long-term debt market because of the prospect of risky asset substitution."
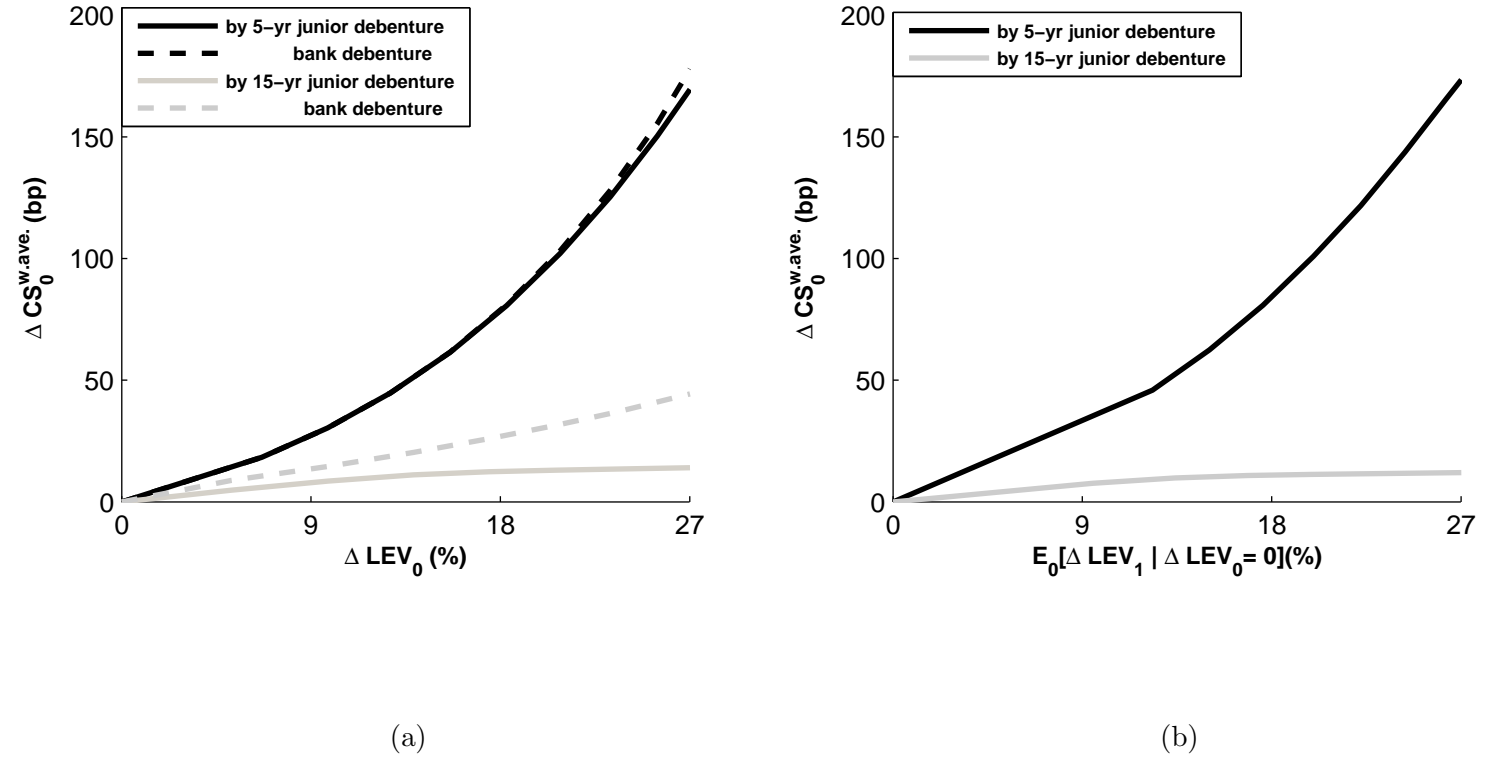
(a)   (b)

Figure 5: The Change of a Firm's Credit Spreads versus the Change of the Current and Future Leverage Ratio. Suppose that a firm's debt structure is comprised by three types of bonds: bank debenture, senior debenture and long-term debenture with 10% coupon paid semi-annually. The face value is 100; the priority rule is bank debenture $\succ$ senior debenture $\succ$ long-term debenture; the remaining maturity of the bank debenture, senior debenture and long-term debenture is 8, 12 and 20 years, respectively. LEV_t represents the firm leverage ratio at time t, defined as $(Total\ Face\ Value\ of\ Debt)_t/[(Total\ Face\ Value\ of\ Debt)_t+(Market\ Value\ of\ Equity)_t]$. $CS_0^{w.ave.}$ represents the weighting average credit spread at $t = 0$; that is, the weighting average credit spread of the bank debenture, senior debenture and long-term debenture. In panel (a), the black and gray solid lines indicate that the leverage ratio is increased by issuing a 5- or 15-year junior debenture (the most subordinated bond); the black and gray dashed lines indicate that the leverage is increased by a 5- or 15-year bank debenture. In panel (b), the black or gray solid line indicates that the leverage is expected to be increased in a year (i.e. at $t = 1$) by issuing a 5- or 15-year junior debenture. The numerical settings are $V_0 = 600$, $r = 6\%$, $\sigma = 0.2$, $\tau = 0.35$ and $\alpha = 0.5$.

quality. Such result captures Ingersoll (1987)'s argument that the relative priority of the SD improves because the BD is replaced but the effective priority of SD deteriorates greatly because the JD matures before the SD. However, in **Fig. 6(b1)** and **(b2)**, if the payment blockage covenant is considered, such dilution effect is greatly attenuated. Moreover, the improvement in the firm's credit reduces the aforementioned difference, leading to the numerical results in accord with Linn and Stock's empirical research.
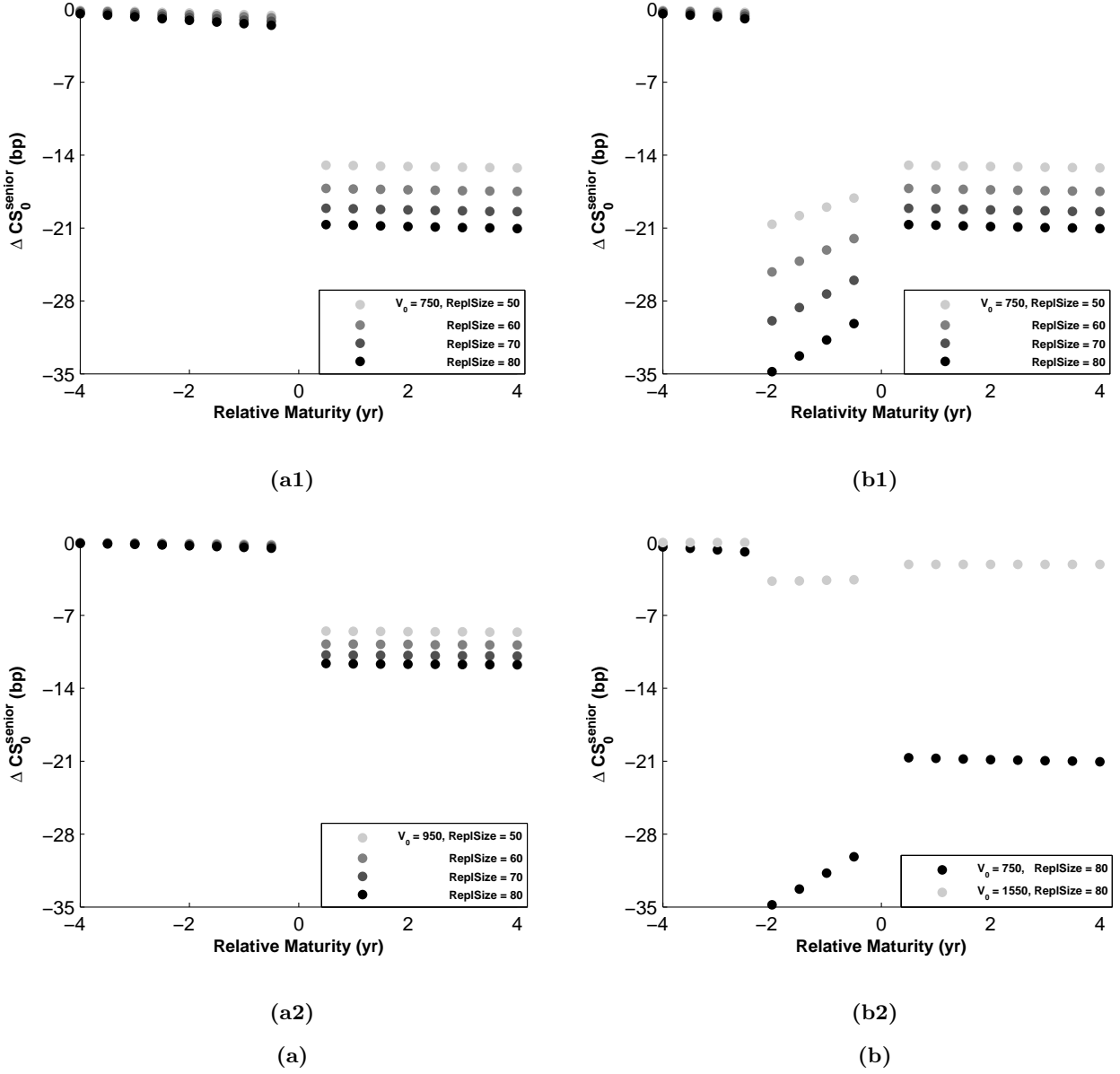
Figure 6: The Impact of a Junior Debenture Issuance to Replace a Bank Debenture on a Senior Debenture. Suppose that a firm's debt structure is comprised by three types of bonds: bank debenture, senior debenture and long-term debenture with 10% coupon paid semi-annually. The total face value of those bonds is 500 and the face value of the long term debenture is 100. The remaining maturity of the senior debenture and long-term debenture are 12 and 20 years. A junior debenture is now issued to replace the existing bank debenture. Panel (a) and (b) display the impact of this junior debenture issuance on the senior debenture without or with payment blockage, given the total face value of the debt unchanged. Blockage period is supposed to be 2 years. The $x-$axis denotes the relative maturity. If the maturity of the junior and senior debenture are 8 and 12 years from now, the relative maturity is -4 years. The $y-$axis denotes the change of spread after the junior debenture is issued to replace the bank debt. The "ReplSize" denotes the face value of the bank debt replaced by the junior debenture. Other numerical settings are $r = 6\%$, $\sigma = 0.2$, $\tau = 0.35$ and $\alpha = 0.5$.

4.2   Call Policy, Market Friction and Maturity Structure

Copious topics about callable bonds are studied in previous research. Great majority of these literature centers on the discussion regarding the rationales of utilizing call provisions and the characteristics of issuers that embed such provisions in bonds. Others follow the argument of Black-Scholes model (Black and Scholes, 1973) to establish an option-based valuation framework for callable bonds (Brennan and Schwartz, 1977; Ingersoll, 1977a). Implied by this methodology, the embedded call options serve to minimize the bond values, and the optimal call policy is that issuers will immediately redeem the bond when its market value exceeds the effective call price.

Given an investment policy, underinvestment problems may arise once levered equity holders forgo part of the projects with positive net present value because they must share almost all the increases in firm value with bond holders. Bodie and Taggart (1978) and Barnea et al. (1980) state that bonds with the feature of callability can alleviate such moral hazard. Acharya and Carpenter (2002) further demonstrate this idea by showing that, when the market is frictionless, callable bonds under the textbook policy can attenuate underinvestment problems because the embedded call provisions can increase equity sensitivity to firm value (denoted as $\partial E/\partial V_0$). Compared with the the equity sensitivity as the firm issue a straight bond (denoted as $\partial E_s/\partial V_0$), higher equity sensitivity (i.e., $\partial E_c/\partial V_0 > \partial E_s/\partial V_0$) represents that, given per unit of increase in firm value, levered equity holders can keep more profits in hands by redeeming bonds to decrease future coupon payments. Hence, bond redemptions increase equity sensitivity at the expense of corresponding bond sensitivity to firm value (i.e., $\partial B_s/\partial V_0 > \partial B_c/\partial V_0$), as shown in **Fig 7. (a)**. However, if callable bonds are still manipulated by the same policy, the presence of market friction, such as corporate income tax, may invalidate the aforementioned underinvestment-mitigation function when the firm's credit deteriorates, as shown in **Fig 7. (b1)**. This suggests that the environment of a capital market and corresponding call policies are the crucial role to put call features into effect. The argument that the textbook policy is optimal to equity holders is based on Modigliani and Miller (1958) capital structure irrelevance theory; that is, the values between equity and bond holders can be modeled by a zero-sum game when the capital market is frictionless. Therefore, the textbook policy can not only minimize the bond value but maximize the equity value. Similarly, to mitigate the underinvestment problem, such policy can raise equity sensitivity to firm value by reducing the corresponding bond sensitivity. However, with the presence of a market friction, the textbook policy is not always optimal to equity holders regarding the mitigation of underinvestment problems. For instance, considering the existence of corporate income tax, reducing debt level through bond redemptions is a tradeoff. On the one hand, the equity sensitivity to firm value is increased due to less future coupon payments but, on the other hand, is decreased due to less tax interest savings. If managers obey assumption **(A.2)**, the optimal call policy should serve to increase equity value and guarantee $\partial E_c/\partial V_0 > \partial E_s/\partial V_0$, as shown in **Fig 7. (b2)**.

The textbook policy is also challenged by observable premiums over the effective call prices (PoCP) in the financial market, because such policy suggests that the market value of a callable bond cannot exceed its effective call price (CP). Positive PoCP implies that the timing of call is delayed compared with the timing implied by the textbook policy. Among the literature that attempts to explain such deviation through the presence of market frictions, Mauer (1993) and Dunn and Spatt (2005) specify refunding transaction costs. Ingersoll (1977b) and Asquith (1995) accentuate corporate income tax. Equity holders may postpone the timing of call to reserve more interest tax savings. Other than market friction, Jones et al. (1984) analytically demonstrate that the textbook policy fails to be optimal to equity holders due to the presence of multiple bond holders. Longstaff and Tuckman (1994) further
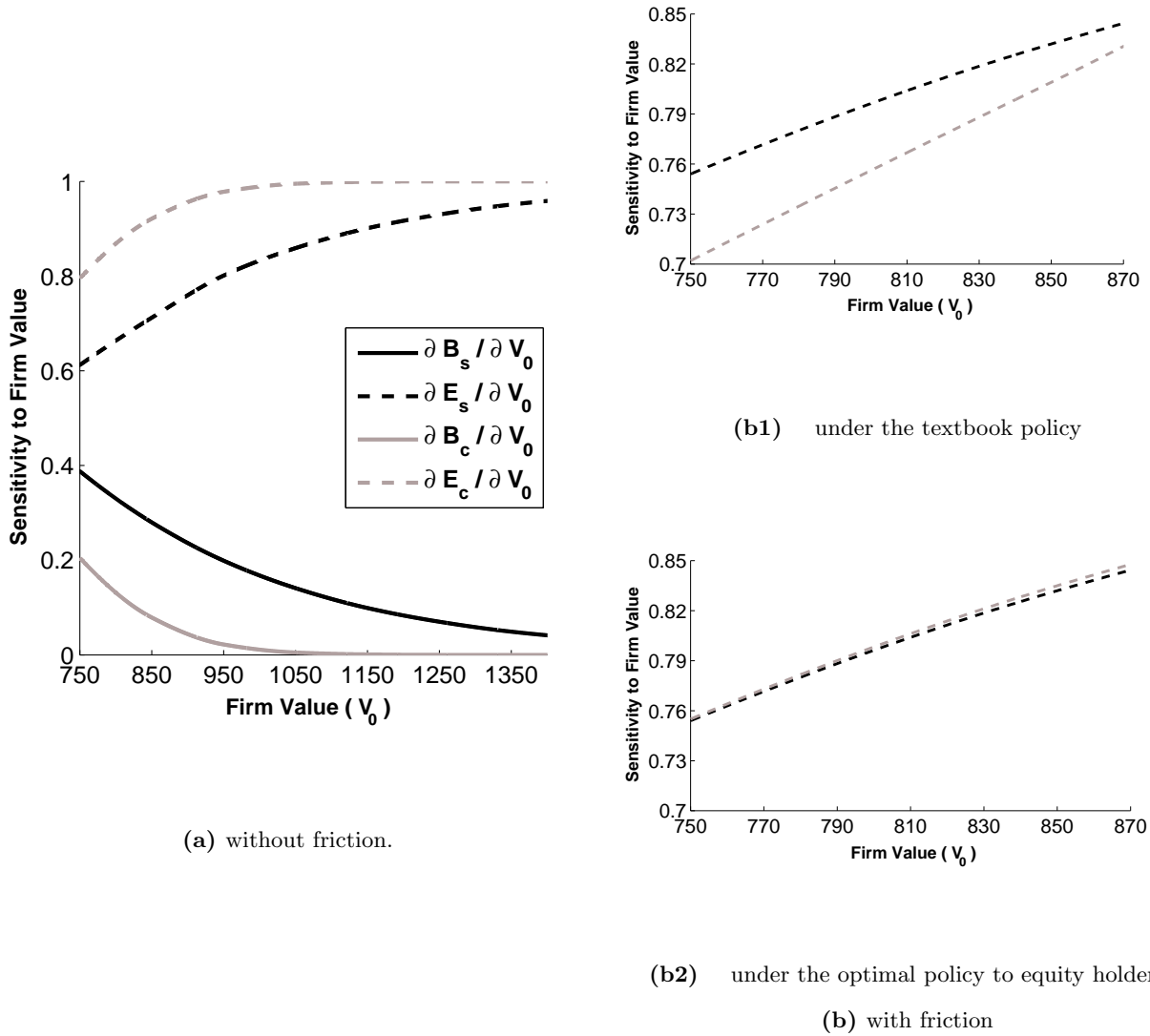
(a) without friction.

(b1) under the textbook policy

(b2) under the optimal policy to equity holders

(b) with friction

Figure 7: On the mitigation of underinvestment problems. Assume a firm issues a 7-year callable bond with face value 600 and coupon rate 10% paid semi-annually. E_s and B_s denote the value of equity and the corresponding straight bond; E_c and B_c denote the value of equity and the corresponding callable bond. The effective call price is the face value plus accrued interest; the call dates are each coupon payment date. The x-axis denotes the initial firm value that is used to proxy a firm's credit quality; the y-axis denotes the equity or bond sensitivity to firm value. In panel (a), $\tau = 0$ and $\alpha = 0$, whereas in panel (b), $\tau = 0.35$ and $\alpha = 0.5$. Other numerical settings are $r = 5.8\%$ and $\sigma = 0.2$.

state that the change of capital structure through bond redemptions may result in wealth redistribution to remaining bond holders so that equity holders delay call to alleviate such wealth transfer effect. **Table 2.** provides a numerical example to illustrate wealth redistribution resulting from a leverage decreasing call.[15] Assume that a firm issues 5 bonds with equal priority, face value 120 and coupon rate 10% paid semi-annually. The time to maturity for those bonds is 3, 5, 7, 9, and 12 years. B_1 to B_5 and E denote the expected present value of each bond and the corresponding equity. $\mathcal{D}_s$ represents the debt structure with all straight bonds; $\mathcal{D}_c^m$ denotes the debt structure with 4 straight bonds and a callable bond B_3 under the textbook policy; $\mathcal{D}_c^M$ denotes the otherwise identical debt structure with

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[15]We follow the characteristics of King and Mauer (2000)'s data set that most calls are leverage decreasing calls.

a callable bond B_3 under the call policy that follows the assumption **(A.2)**. The effective call price is the face value plus accrued interest($120 + 6 = 126$). If managers follow the textbook policy, the value of the callable bond B_3 does not exceed its effective call price ($122.40 < 126$). Besides, compared with the equity value with respect to the debt structure $\mathcal{D}_s$, potential change of capital structure through the bond redemption redistribute most part of wealth to bond B_4 and B_5 and decrease the equity value. That is because the leverage decreasing call reduces the issuer's default risk (or the default risk of bond B_4 and B_5) and the remaining bond holders will be compensated prior to equity holders. The worse the issuer's credit, the more the remaining bond holders absorb the compensation, leading to less equity value. Obviously, the textbook policy is not optimal to equity holders under this situation.[16] If managers follows the call policy that maximizes equity holders benefits, they postpone the timing of call until little compensation is shared by the remaining bond holders (i.e., the values of B_4 and B_5 in $\mathcal{D}_c^M$ are less than that in $\mathcal{D}_c^m$). Note that the value of B_3 in the debt structure $\mathcal{D}_c^M$ is greater than its effective call price, and this premium is the proxy to measure the extent of call delay.

V_0		1100			1300			1500		
		$\mathcal{D}_s$	$\mathcal{D}_c^m$	$\mathcal{D}_c^M$	$\mathcal{D}_s$	$\mathcal{D}_c^m$	$\mathcal{D}_c^M$	$\mathcal{D}_s$	$\mathcal{D}_c^m$	$\mathcal{D}_c^M$
B_1	(3 yrs)	133.40	133.40	133.40	133.40	133.40	133.40	133.40	133.40	133.40
B_2	(5 yrs)	141.07	141.03	141.07	141.11	141.11	141.11	141.12	141.12	141.12
B_3	(7 yrs)	146.66	**122.40**	**139.05**	147.59	**122.40**	**134.11**	147.87	**122.40**	**128.54**
B_4	(9 yrs)	149.10	151.90	149.15	152.03	153.42	152.13	153.23	153.89	153.38
B_5	(12 yrs)	152.53	156.10	152.71	157.41	159.59	157.74	159.79	161.00	160.19
E		491.56	487.12	491.77	684.66	683.59	685.12	881.72	881.93	882.52

Table 2: **The Wealth Transfer Effect.** Assume a firm issues 5 bonds with equal priority. The face value and coupon rate of those bonds are 120 and 10% paid semi-annually, respectively. The time to maturity for those bonds is 3, 5, 7, 9, and 12 years. B_1 to B_5 and E denote the value of each bond and the corresponding equity. $\mathcal{D}_s$ represents the debt structure with all straight bonds. $\mathcal{D}_c^m$ and $\mathcal{D}_c^M$ represents the debt structure with 4 straight bonds and a callable bond B_3 under the textbook policy and under the policy that maximizes the equity benefits, respectively. The effective call price is the face value plus accrued interest; the call dates are each coupon payment date. Other numerical settings are $r = 5.8\%$, $\sigma = 0.2$, $\tau = 0.35$ and $\alpha = 0.5$.

The presences of tax interest savings and the wealth transfer effect are two possible reasons we emphasize to explain the "call delay phenomenon". To ensure that call decisions can maximize equity holders' benefits, managers have incentive to wait longer until they reserve enough tax interest savings or until the wealth transfer effect becomes moderate (i.e., when the firm's credit becomes better). **Fig. 8 (a)** displays the connection between the two reasons under the call policy that maximizes equity holders' benefits. The case of single bond with tax (plotted in the dot-dashed curve) exhibits positive normalized PoCP (PoCP/CP) and the hump-shaped relationship between normalized PoCP and firm value. The normalized PoCP becomes greater under the multiple-bond case (plotted in the thick solid curve), suggesting that the wealth transfer effect magnifies the call delay phenomenon due to the presence of corporate income tax. Indeed, there exists samples with normalized PoCP greater than 10% in King and Mauer (2000)'s data set.[17] Longstaff and Tuckman (1994) explain this hump-

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[16]Actually, Longstaff and Tuckman (1994) state that "the textbook policy is optimal only if the firm's capital structure does not change in the process of calling a bond issue". That is, "the bond has to be refunded with an issue that has exactly the same remaining interest payments, sinking fund provisions, and option features as the original issue." However, "many callable bonds are not refundable."

[17]In their sample, more than 86% of the firms redeem bonds long after the market value of the bond first reaches the call price. The average call delay is over 27 months.

shaped relationship as follows: A callable bond first appreciates with the deterioration in the firm's credit since the manager may waits until the wealth transfer effect becomes moderate; the bond then depreciates due to the default risk if the firm's credit deteriorates further. That is, default risk and the issuer's redemption strategy (considering both wealth transfer effect and tax benefits) alternatively alternatively dominate the determination of callable bond prices.

King and Mauer (2000) empirically reject the impact of wealth transfer effect on the call delay phenomenon by directly showing that the hump-shaped relationship between normalized `PoCP` and a firm's credit is insignificant. Instead, their sample exhibits the pattern that, other things being equal, normalized `PoCP` would significantly increase with the improvement in a firm's credit (i.e., a monotonic relationship between normalized `PoCP` and firm value) and decrease with the level of interest rate. Conceptually, these are the features in which a straight bond is inherent (i.e., the characteristics of the callable bonds in King and Mauer's sample are similar to that of straight bonds).[18] Motivated by this observation, the factors which make callable and straight bonds different may bridge the gap between Longstaff and Tuckman (1994) and King and Mauer (2000). The first intuitive factor is call protection. Other things being equal, callable bonds become more similar to straight bonds the longer the call protection period. As the numerical results in **Fig. 8 (b1)**, the solid lines display the hump-shaped pattern that Longstaff and Tuckman claim to explain the call delay phenomenon due to wealth transfer effect. However, if the call protection period is longer enough, the relationship between normalized `PoCP` and firm value changes from hump-shaped to monotonic (plotted in the dashed curve). Actually, we find that, in King and Mauer's sample,[19] the call protection period on average accounts for more than 65% (6.41 years out of 9.83 years) of the original maturity on callable bonds. The second factor is the level of interest rate. Other things being equal, callable bonds become more similar to straight bonds the higher the level of interest rate. We observe the variation in the level of interest rate during the periods studied by Longstaff and Tuckman (1994) and King and Mauer (2000). The samples covered by the former study is from August 1991 to August 1992, whereas the latter is covered from January 1975 to March 1994, as shown in **Fig. 9**. The average level of interest rate for the former short period, 7.31%, is significantly lower than that for the latter long period, 9.22%. The numerical results in **Fig. 8 (b2)** exhibit that, other things being equal, the relationship between normalized `PoCP` and firm value again changes from hump-shaped (plotted in the solid curve) to monotonic (plotted in the dashed curve) as the level of interest rate increases. Besides, the variation from hump-shaped to monotonic is very sensitive to the level of interest rate. Thus, the level of interest rate can explain the conflicts between the two studies. In summary, the value of callable bonds reflects issuers' call policies in different situations, and that makes the relationship between normalized `PoCP` and firm value alternate between hump-shaped and monotonic.

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[18]A callable bond can be regarded as a straight bond minus a call option. Given a coupon rate and call price, the value of the embedded call option decreases with the value of `PoCP`. The `PoCP` implied by the trivial call option value is equivalent to the `PoCP` for a straight bond. Thus, the `PoCP` for a straight bond increases with the improvement in a firm's credit and decreases with the level of interest rate.

[19]The information about the call protection period refers to King (2002). Most part of the samples in King (2002) are identical to that in King and Mauer (2000), but the two studies concentrate on different topics.
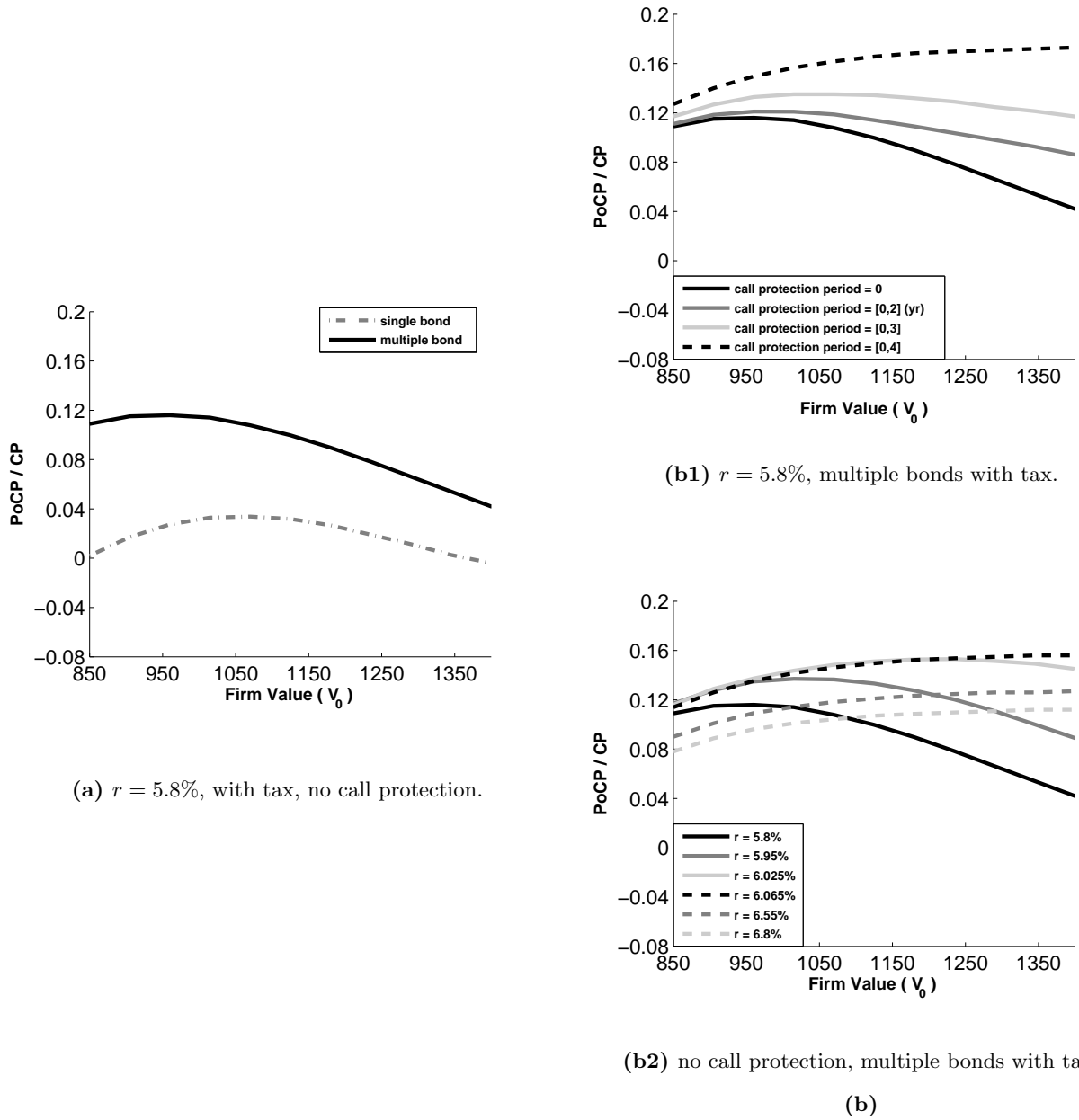
(a) $r = 5.8\%$, with tax, no call protection.

(b1) $r = 5.8\%$, multiple bonds with tax.

(b2) no call protection, multiple bonds with tax.

(b)

Figure 8: Premiums over Effective Call Prices. Premiums over effective call prices (PoCP) are defined as the price of a callable bond minus the effective call price (CP). In panel (a), the term "single bond" denotes the case that a firm issues a 7-year 10% callable bond with face value 600; "multiple bonds" represents the case that a firm issues 5 bonds — 4 straight bonds and 1 callable bond — with equal priority. The face value and coupon rate of those bonds are 120 and 10%, respectively. The time to maturity for those straight bonds is 3, 5, 9, 12 years, and the maturity for the callable bond is 7 years. In panel (b1), PoCP are evaluated in multiple-bond case under different call protection period given interest rate level. In panel (b2), PoCP are evaluated in multiple-bond case under different interest rate levels given no protection period. The x-axis denotes the initial firm value that is used to proxy a firm's credit quality; the y-axis denotes PoCP/CP which is a normalized measure of call delay. Other numerical settings are $\sigma = 0.2$, $\tau = 0.35$ and $\alpha = 0.5$.
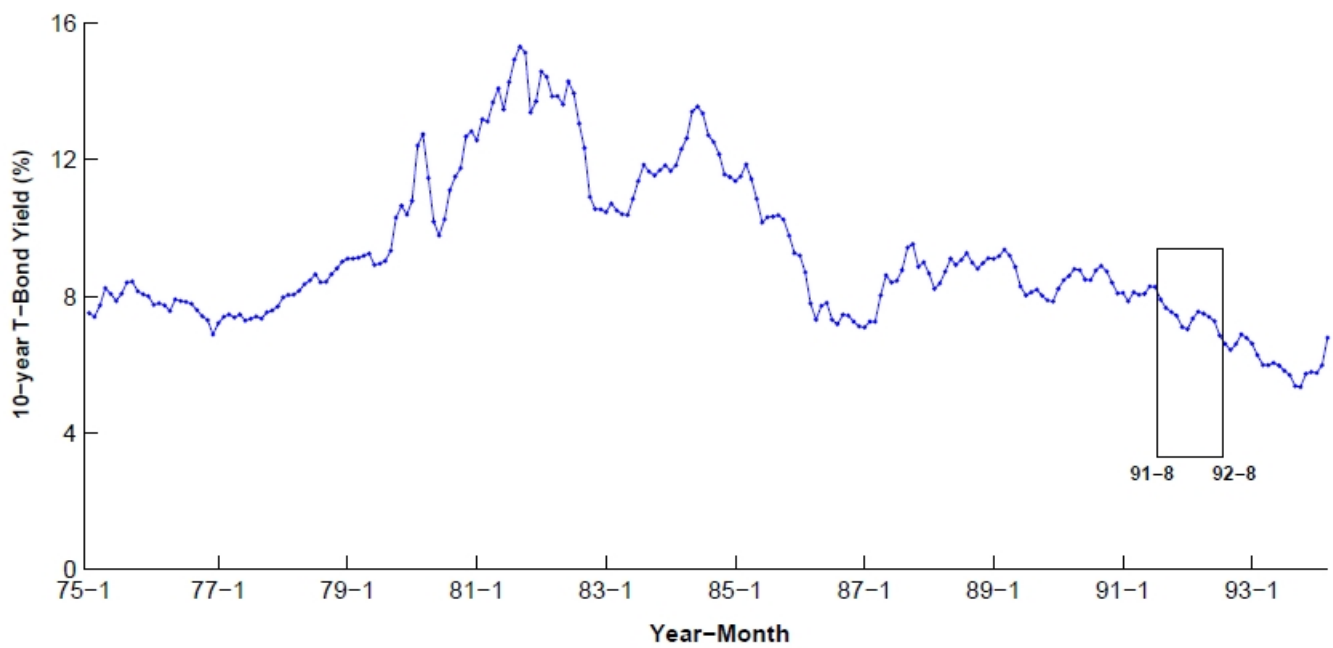
Figure 9: 10-year treasury-bond yields. 10-year treasury yields are used as proxy of interest rate level. The whole graph plot the yields for the sample period of King and Mauer (2000); the samples highlighted in the rectangle are the period of Longstaff and Tuckman (1994). The average yields in the whole and highlighted period is 9.22% and 7.31%, respectively.

4.3 Poison Put Covenant

During the period between 1983 and 1988, about 10% of corporate bonds were downgraded from investment- to speculative-grade due to the booming of financial restructuring, such as leveraged buyouts (LBO) and leveraged recapitalization. To alleviate bond holders' concern for such marked credit deterioration and the corresponding leverage-induced loss, event-risk covenants, such as poison puts, are introduced in the corporate bond markets to prevent such type of bold decisions made by levered equity holders. As noted by Crabbe (1991), almost half of the investment-grade new offerings were devise event-risk covenants during the period from November 1988 through December 1989. the statistics from Billett et al. (2007) further state the percentage of bonds that devise poison puts increases from 15.8% in 1985-1989 to 37.8% in 1995-1999. Due to the prevalence, various investigations are conducted. Cook and Easterwood (1994) analyzes the economic role of poison puts and empirically reports that such covenants have negative effect on equity holders and positive effect on outstanding bond holders. This results can be directly explained through the nature of poison puts that serve to impose additional costs on prospective bids, which may benefit equity holders but impair the target bond holders. Cremers et al. (2007) further express that bond holders governance through the use of poison puts can mitigate the asset substitution problem and make the interest of equity and target bond holders relatively accordant.

As the description in bond contracts, poison puts grant bond holders to put the bonds back to the firm at par or at premiums under certain specified conditions. Such conditions could be the occurrence of financial restructuring, downgrade from investment to speculative grade or the leverage elevation to specified levels. Taking a LBO for example, reducing asset substitution problem by using poison puts can be interpreted in two ways. First, the potential trigger of put deters prospective bidders because their costs of obtaining additional debt capitals are increased. The failure in the LBO favors the target bond holders, but equity holders lose the deal that may be risky but profitable to them. Second, a bidder accomplishes the LBO despite the cost of newly issued bonds, which usually have higher priority than the bonds in target firms. The achievement in LBO may benefit equity holders, but part of their wealth are retrieved by target bond holders as compensation through the trigger of put. Our numerical model can present the latter interpretation. Suppose that a bidder will decide to issue 4 equally priority bonds with face value 50 and maturity 3, 5, 7 and 10 years to accomplish a LBO. The face value, coupon rate and maturity of the target firm's bond are 250, 10% and 20 years, respectively. Besides, those newly issued bonds is senior than the bond in the target firms. The value of the bidder's firm after the LBO is 1400, and the poison puts devised in the target firm's bond will be triggered once the bidder's firm asset value falls below its total bond face value.[20] Given protection levels, the corresponding costs for newly issued bonds are displayed in **Fig 10 (a)**. Potential trigger of the poison put raises the costs of newly issued bonds, because their effective priority will be deteriorated due to the potential prepayment to the target bond holders. **Fig 10 (b)** exhibits that, compared with the case that without poison puts, the more the put price the more the target bond holders are compensated. The wealth of equity holders (or the bidder) are deprived due to the higher financing costs for the LBO and higher default risk because the target bond holders are prepaid. These numerical results are implicitly supported by the conclusions of Cremers et al. (2007) that the increase in yield spreads of the bonds without such covenant protections is much greater than that with covenant protections when the target bond holders are exposed to event risks.

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[20]Our numerical model can also simulate the case that the poison put covenants are triggered once the LBO are accomplished. The results will be similar.
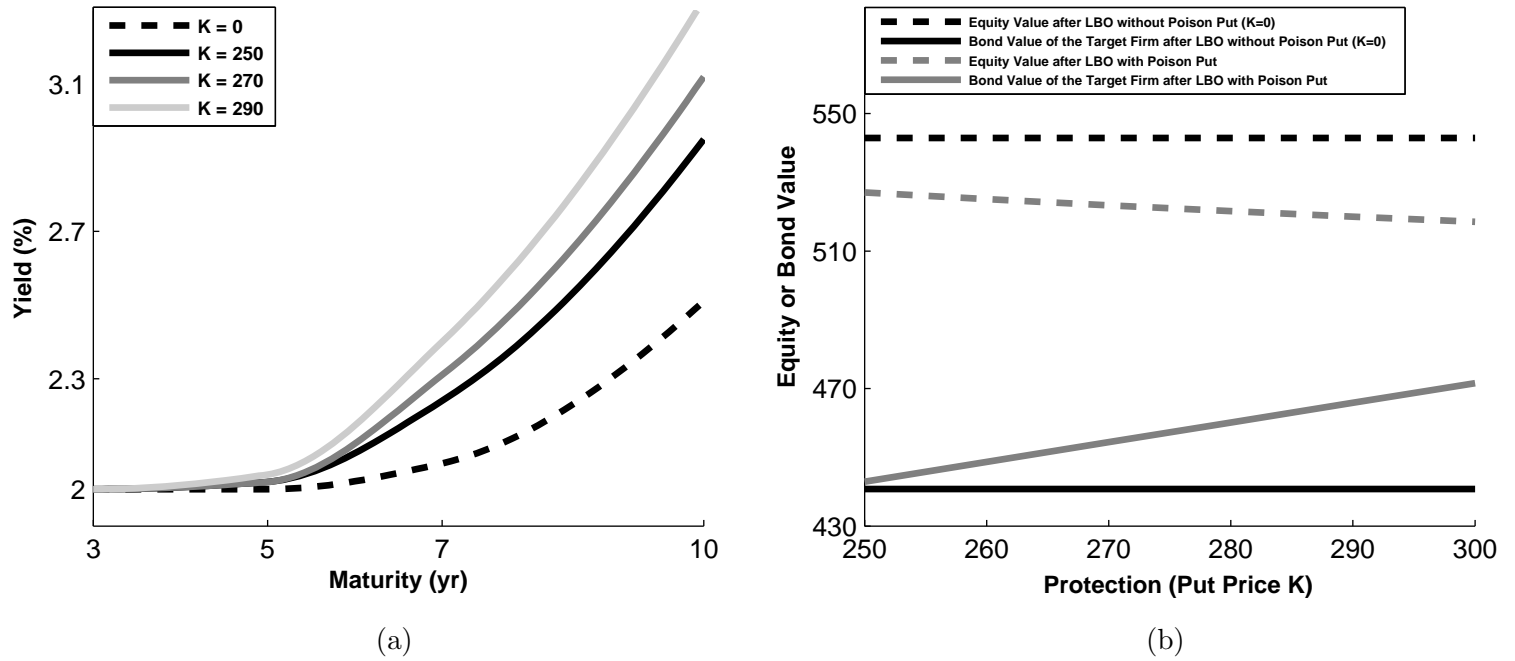
Figure 10: Bond value of the target firm, cost of new issued debt and equity value after leveraged buyout. Suppose a firm accomplishes a leveraged buyout by issuing 4 debts with face value 50 and maturity 3, 5, 7 and 10 years. The priority of these new issued debts are higher than the target firm's debt. The face value, coupon rate and maturity of the target firm's debt are 250, 10% and 20 years, respectively. Panel (a) displays the yield of those new issued bonds for leveraged buyout corresponding to a given protection level for target firm's debt. The poison put covenant will be triggered once the firm value after leveraged buyout falls below the boundary contracted beforehand. Here the boundary is set to be the lump sum of the firm's debt face value. The protection for the target firm's debt is measured by the put price K. Panel focuses on the value of equity and the target firm's debt with or without poison put covenants after leveraged buyout. The firm value after leveraged buyout $V_0 = 1400$ and $\sigma = 0.25$. Other numerical settings are $r = 2\%$, tax rate $\tau = 0.35$ and bankruptcy cost $\alpha = 0.5$.

5   Conclusion

If the market participants recognize the heterogeneity of corporate debt structure, then it is nature to anticipate that market prices would account for this fact. Selectively simplifying this recognition probably leads to biased estimations. This article develops a theoretical framework to investigate the effects of heterogeneous debt structure on the values of a firm's claim holders from four observable facets of corporate debt structure: the leverage ratio, maturity structure, priority structure and covenant structure. Besides, we propose a novel numerical method to quantitatively analyze the behaviors of the claim holders that are granted the rights to earlier redeem their bonds when the corporate debt structure is complicated. Compared with extant credit risk models, the results generated by our framework are more consistent with the real world phenomena documented already in empirical studies. First, our framework robustly generate upward-sloping credit spread curves for either investment- or speculative-grade issuers as the observation by Helwege and Turner (1999) and Huang and Zhang (2008), whereas numerous existing structural models may predict hump-shaped or downward-sloping curves. Second, our framework not only predicts a positive correlation between a firm leverage ratio and firm credit quality but also forecasts a negative relation between bond credit spreads and the improvement in bond seniorities. If a bank debenture is replaced by a junior debenture issuance, the credit spreads of the previously issued senior debenture may decrease because its relative priority in the firm's debt structure improves. Moreover, it predicts weak negative impacts of the presence of short-term junior debenture on the prices of the existing long-term senior debenture when the issuing firm is relatively healthy because there exists payment blockage covenants in the senior debenture, but such impacts become significantly negative when the firm's credit deteriorates because the blockage period is limiting. Incorporating payment blockage into our framework makes our analyses regarding the change of both maturity and priority structure in compliance with the empirical observation as Linn and Stock (2005). Third, our framework predicts longer call delay because we jointly consider the impacts of tax shield benefits and the wealth transfer effects on the firm's optimal redemption policy. We find that the wealth transfer effect may magnify the call delay phenomenon due to the presence of tax shield benefits. Furthermore, we explains the conflicts between the theoretical predictions and the empirical observations of the wealth transfer effects through the level of interest rate and the existence of call protection covenants. Finally, our framework displays the salient impact of including poison put covenants on the value of target bond holders and a potential bidder's costs of debt financing. It thus appears that accounting for the heterogeneity of corporate debt structure has a significant impact on corporate bond valuation, and it helps to reconcile many of the predictions of extant structural models with empirical observations.

References

Acharya, v. and J. Carpenter (2002). Corporate bond valuation and hedging with stochastic interest rates and endogenous bankruptcy. *Review of Financial Studies 15*(5), 1355–1383.

Altman, E. I. (1968). Financial ratios, discriminant analysis and the prediction of corporate bankruptcy. *Journal of Finance 23*(4), 589–609.

Asquith, P. (1995). Convertible bonds are not called late. *Journal of Finance 50*(4), 1275–1289.

Attaoui, S. and P. Poncet (2013). Capital structure and debt priority. *Financial Management.*

Barnea, A., R. A. Haugen, and L. W. Senbet (1980). A rationale for debt maturity structure and call provisions in the agency theoretic framework. *Journal of Finance 35*(5), 1223–1234.

Billett, M. T., T.-H. D. King, and D. C. Mauer (2007). Growth opportunities and the choice of leverage, debt maturity, and covenants. *Journal of Finance 62*(2), 697–730.

Black, F. and J. Cox (1976). Valuing corporate securities: Some effects of bond indenture provisions. *Journal of Finance 31*(2), 351–367.

Black, F. and M. Scholes (1973). The pricing of options and corporate liabilities. *Journal of Political Economy 81*(3), 637.

Bodie, Z. and R. A. Taggart (1978). Future investment opportunities and the value of the call provision on a bond. *Journal of Finance 33*(4), 1187–1200.

Brennan, M. J. and E. S. Schwartz (1977). Savings bonds, retractable bonds and callable bonds. *Journal of Financial Economics 5*(1), 67–88.

Bris, A., I. Welch, and N. Zhu (2006). The costs of bankruptcy: Chapter 7 liquidation versus chapter 11 reorganization. *Journal of Finance 61*, 1253–1303.

Briys, E. and F. De Varenne (1997). Valuing risky fixed rate debt: An extension. *Journal of Financial and Quantitative Analysis 32*(2), 239–248.

Broadie, M. and O. Kaya (2007). A binomial lattice method for pricing corporate debt and modeling chapter 11 proceedings. *Journal of Financial and Quantitative Analysis 42*, 1253–1303.

Colla, P., F. Ippolito, and K. Li (2013). Debt specialization. *Journal of Finance.*

Collin-Dufresne, P. and R. S. Goldstein (2001a). Do credit spreads reflect stationary leverage ratios? *Journal of Finance 56*(5), 1929–1957.

Collin-Dufresne, P., R. S. Goldstein, and J. S. Martin (2001b). The determinants of credit spread changes. *Journal of Finance 56*(6), 2177–2207.

Cook, D. O. and J. C. Easterwood (1994). Poison put bonds: An analysis of their economic role. *Journal of Finance 49*(5), 1905–1920.

Cox, J. C., S. A. Ross, and M. Rubinstein (1979). Option pricing: A simplified approach. *Journal of Financial Economics 7*(3), 229–263.

Crabbe, L. (1991). Event risk: An analysis of losses to bondholders and super poison put bond covenants. *Journal of Finance 46*(2), 689–706.

Crabbe, L. E. and J. Helwege (1994). Alternative tests of agency theories of callable corporate bonds. *Financial Management 23*(4), 3–20.

Cremers, K. M., V. B. Nair, and C. Wei (2007). Governance mechanisms and bond prices. *Review of Financial Studies 20*(5), 1359–1388.

Cummings, N., J. Thompson, and V. Paparo (January, 2009). Back to basics bnking. *International Financial Law Review*.

Dai, T. and Y. Lyuu (2010). The bino-trinomial tree: A simple model for efficient and accurate option pricing. *Journal of Derivatives 17*(4), 7–24.

Dai, T.-S., C.-J. Wang, and Y.-D. Lyuu (2012). A multiphase, flexible, and accurate lattice for pricing complex derivatives with multiple market variables. *Journal of Futures Markets*.

Davydenko, S. (2007). When do firms default? a study of the default boundary. *American Finance Association 2009 Annual Meeting*.

Diamond, D. W. (1993). Seniority and maturity of debt contracts. *Journal of Financial Economics 33*(3), 341–368.

Duffie, D. (1996). *Dynamic asset pricing theory*. Princeton University Press.

Duffie, D. and K. J. Singleton (1999). Modeling term structures of defaultable bonds. *Review of Financial Studies 12*(4), 687–720.

Dunn, K. B. and C. S. Spatt (2005). The effect of refinancing costs and market imperfections on the optimal call strategy and the pricing of debt contracts. *Real Estate Economics 33*(4), 595–617.

Eom, Y. H., J. Helwege, and J.-z. Huang (2004). Structural models of corporate bond pricing: An empirical analysis. *Review of Financial Studies 17*(2), 499–544.

Fama, E. F. and M. H. Miller (1972). *The Theory of Finance*, Volume 3. Dryden Press Hinsdale, IL.

Fan, H. and S. M. Sundaresan (2000). Debt valuation, renegotiation, and optimal dividend policy. *Review of Financial Studies 13*(4), 1057–1099.

Figlewski, S. and B. Gao (1999). The adaptive mesh model: a new approach to efficient option pricing. *Journal of Financial Economics 53*(3), 313–351.

Fisher, L. (1984). Contingent claims analysis of corporate capital structures: An empirical investigation: Discussion. *Journal of Finance 39*(3), 625–627.

Flannery, M. J., S. S. Nikolova, and Ö. Öztekin (2012). Leverage expectations and bond credit spreads. *Journal of Financial and Quantitative Analysis 47*(04), 689–714.

Geske, R. (1977). The valuation of corporate liabilities as compound options. *Journal of Financial and Quantitative Analysis 12*(04), 541–552.

Gryglewicz, S. (2011). A theory of corporate financial decisions with liquidity and solvency concerns. *Journal of Financial Economics 99*(2), 365–384.

Guedes, J. and T. Opler (1996). The determinants of the maturity of corporate debt issues. *Journal of Finance 51*(5), 1809–1833.

Hackbarth, D. and D. C. Mauer (2012). Optimal priority structure, capital structure, and investment. *Review of Financial Studies 25*(3), 747–796.

Harrison, J. M. and D. M. Kreps (1979). Martingales and arbitrage in multiperiod securities markets. *Journal of Economic Theory 20*(3), 381–408.

He, Z. and W. Xiong (2012). Rollover risk and credit risk. *Journal of Finance 67*(2), 391–430.

Helwege, J. and C. M. Turner (1999). The slope of the credit yield curve for speculative-grade issuers. *Journal of Finance 54*(5), 1869–1884.

Huang, J.-z. and M. Huang (2012). How much of the corporate-treasury yield spread is due to credit risk? *Review of Asset Pricing Studies 2*(2), 153–202.

Huang, J.-Z. and X. Zhang (2008). The slope of credit spread curves. *Journal of Fixed Income 18*(1), 56–71.

Hull, J. (2012). *Options, Futures, and Other Derivatives, 8/e.* Pearson Education India.

Ingersoll, J. E. (1977a). A contingent-claims valuation of convertible securities. *Journal of Financial Economics 4*(3), 289–321.

Ingersoll, J. E. (1977b). An examination of corporate call policies on convertible securities. *Journal of Finance 32*(2), 463–478.

Ingersoll, J. E. (1987). *Theory of financial decision making*, Volume 3. Rowman & Littlefield.

Jarrow, R. A., D. Lando, and S. M. Turnbull (1997). A markov model for the term structure of credit risk spreads. *Review of Financial Studies 10*(2), 481–523.

Jarrow, R. A. and S. M. Turnbull (1995). Pricing derivatives on financial securities subject to credit risk. *Journal of Finance 50*(1), 53–85.

Jensen, M. C. and W. H. Meckling (1976). Theory of the firm: Managerial behavior, agency costs and ownership structure. *Journal of Financial Economics 3*(4), 305–360.

Jones, E. P., S. P. Mason, and E. Rosenfeld (1984). Contingent claims analysis of corporate capital structures: An empirical investigation. *Journal of Finance 39*(3), 611–625.

Ju, N. and H. Ou-Yang (2006). Capital structure, debt maturity, and stochastic interest rates*. *Journal of Business 79*(5), 2469–2502.

Jung, M., R. B. Ellis, and M. Epplin (2012). Do call provisions reduce agency costs of debt irrespective of maturity and seniority? *Mustang Journal of Accounting and Finance 2*, 36.

Kim, I., K. Ramaswamy, and S. Sundaresan (1993). Does default risk in coupons affect the valuation of corporate bonds?: A contingent claims model. *Financial Management 22*(3), 117–131.

King, T.-H. D. (2002). An empirical examination of call option values implicit in us corporate bonds. *Journal of Financial and Quantitative Analysis 37*(04), 693–721.

King, T.-H. D. and D. C. Mauer (2000). Corporate call policy for nonconvertible bonds*. *Journal of Business 73*(3), 403–444.

Kisgen, D. J. (2006). Credit ratings and capital structure. *Journal of Finance 61*(3), 1035–1072.

Kish, R. J. and M. Livingston (1992). Determinants of the call option on corporate bonds. *Journal of Banking and Finance 16*(4), 687–703.

Lando, D. (2004). *Credit risk modeling: theory and applications.* Princeton University Press.

Leland, H. E. (1994). Corporate debt value, bond covenants, and optimal capital structure. *Journal of Finance 49*(4), 1213–1252.

Leland, H. E. and K. B. Toft (1996). Optimal capital structure, endogenous bankruptcy, and the term structure of credit spreads. *Journal of Finance 51*(3), 987–1019.

Linn, S. C. and D. R. Stock (2005). The impact of junior debt issuance on senior unsecured debts risk premiums. *Journal of Banking and Finance 29*(6), 1585–1609.

Longstaff, F. A. and E. S. Schwartz (1995). A simple approach to valuing risky fixed and floating rate debt. *Journal of Finance 50*(3), 789–819.

Longstaff, F. A. and B. A. Tuckman (1994). Calling nonconvertible debt and the problem of related wealth transfer effects. *Financial Management 23*(4), 21–27.

Madan, D. B. and H. Unal (1998). Pricing the risks of default. *Review of Derivatives Research 2*(2-3), 121–160.

Mauer, D. C. (1993). Optimal bond call policies under transactions costs. *Journal of Financial Research 16*(1), 23–37.

Merton, R. C. (1974). On the pricing of corporate debt: The risk structure of interest rates*. *Journal of Finance 29*(2), 449–470.

Modigliani, F. and M. H. Miller (1958). The cost of capital, corporation finance and the theory of investment. *American Economic Review 48*(3), 261–297.

Myers, S. C. (1977). Determinants of corporate borrowing. *Journal of Financial Economics 5*(2), 147–175.

Park, C. (2000). Monitoring and structure of debt contracts. *Journal of Finance 55*(5), 2157–2195.

Rauh, J. D. and A. Sufi (2010). Capital structure and debt structure. *Review of Financial Studies 23*(12), 4242–4280.

Sarkar, S. (2004). Yield spreads, agency costs and the corporate bond call feature. *European Journal of Finance 10*(4), 308–327.

Sinkey, J. (2002). *Commercial Bank Financial Management in the Financial Services Industry.* New York: Macmillan.

Smith, C. W. and J. B. Warner (1979). On financial contracting: An analysis of bond covenants. *Journal of Financial Economics 7*(2), 117–161.